Homework 8 – due 04/09/08

Math 601

38. Let K be a field of characteristic $p > 0$, and $a \in K$. Prove that if $X^p - X - a$ has no root in K, it is irreducible in $K[X]$.

39. Dummit-Foote, 14.3, #8.

40. Dummit-Foote, 14.2, #3.

41. Dummit-Foote, 14.2, #6.

42. Let p be an odd prime, ξ a primitive pth root of 1 in $\mathbb{C}$ and $K = \mathbb{Q}(\xi)$. We know that $\mathrm{Aut}(K) = (\mathbb{Z}/p\mathbb{Z})^\times \cong \mathbb{Z}/(p-1)\mathbb{Z}$. Thus K has exactly one subfield M with $[M : \mathbb{Q}] = 2$. Show that $M = \mathbb{Q}(\sqrt{\pm p})$ according as $p \equiv \pm 1\,(4)$.