

## Math 603 – Commutative Algebra – Fall 2003

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**Office Hours:** Monday 11-12 am, Friday 11:00 -11:30 am, 2-3 pm, or by appointment.

**Time and Location:** MWF 1:00-1:50 pm, in MTH 0103.

**Text:** M. Atiyah and I. Macdonald, *Introduction to Commutative Algebra*, published by Westview Press (Perseus Books Group), ISBN 0201407515.

### Course Plan:

We will cover commutative algebra along with some related topics in affine algebraic geometry. We will cover as much as possible from the following list of topics:

1. Basic notions: ring homomorphisms, ideals, modules, fundamental theorems
2. Prime and maximal ideals; the nilradical and Jacobson radical; Nakayama's Lemma
3.  $\text{Spec}(A)$ ; the Zariski topology
4. Localization; local properties such as flatness
5. Schemes: Affine schemes; Sheaves, locally ringed spaces, and schemes
6. Integral extensions: Integral dependence; The going-up and going-down theorems; Normality
7. Noether Normalization and its consequences: Hilbert's Nullstellensatz, geometric meaning thereof; Transcendental dimension
8. Noetherian rings: Hilbert Basis Theorem; Discrete valuation rings
9. Completions: Artin-Rees lemma; For Noetherian rings and modules; Associated graded rings
10. Dimension Theory: For Noetherian local rings; Krull's principal ideal theorem; Regular local rings; Normal irreducible curves are smooth; Normality in codimension 1
11. Formally étale and smooth maps; Jacobian criterion; Cohen-Macaulay modules and complete intersections
12. Affine Group schemes: applications to, and proof that they are smooth in characteristic zero.

*Note:* Not all of this material is contained in Atiyah-Macdonald, but most of it is.

### Grading policy:

To get an "A" in this course, you must

- 1) Attend the lectures;
- 2) Make a solid effort on the homework assignments.

The homework assignments will be relatively infrequent: only 3 or 4 during the whole term, approximately one at the end of each month. The assignments will each have approximately 5 problems. Instead of awarding a numerical value, I will mark each assignment with one of the three symbols  $\checkmark+$ ,  $\checkmark$ , or  $\checkmark-$ , and I will write some comments on the papers. (The main purpose of my grading will be to give you feedback, not a numerical grade).

There will be no in-class exams of any kind.

A grade of less than “A” will result if you fail to attend the course regularly and/or make a less than serious effort to do the homework (i.e., you want to avoid getting a  $\checkmark-$ ). If you must miss class for an extended period or cannot return a homework assignment, contact me if you still want an “A” (if truly necessary, I will arrange to give you a ten-minute oral exam to convince me you have learned something about the missed material, thus salvaging the desired “A”).

The homework will appear on the course web-site (see <http://www.math.umd.edu/~tjh>). You should consult that web-site regularly for other announcements related to this course as well.

*Remark:* I recommend that you do as many problems from Atiyah-Macdonald as you can – there are many good ones, and you could while away many a Saturday night working through them all. I will make some effort to point out the especially significant ones (which don’t actually appear in the assigned problems).