Homework $1 - due \ 10/06/05$

Math 603

Do as many as you can!

1. Atiyah-Macdonald, Exercise 21, Chapter 1.

2. Atiyah-Macdonald, Exercise 16, Chapter 3.

3. Let k be any algebraically closed field, and let B be a domain which is a finitely generated k-algebra. Let x be a closed point of Spec(B), and let $U \subset \text{Spec}(B)$ be a non-empty open set. Show that there is a curve in Spec(B) joining x to some point in U. (Note: for the purposes of this exercise, a curve in Spec(B) is a closed set of form $V(\mathfrak{p})$, \mathfrak{p} a prime ideal, where $\dim(B/\mathfrak{p}) = 1$. Also, to avoid trivial counterexamples, we need to assume B is not a field.) Hint: do it first in the case $B = k[X_1, \ldots, X_n]$, then reduce to this case by using the Noether Normalization and the Going-Down theorem.

- 4. Exercise 4.3.3 from the notes.
- 5. Exercise 6.3.3 from the notes.
- 6. Exercise 7.6.2 from the notes.