Math 603 – Commutative Algebra – Fall 2005

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References:

• M. Atiyah and I. Macdonald, *Introduction to Commutative Algebra*, published by Westview Press (Perseus Books Group), ISBN 0201407515.

• D. Eisenbud, Commutative Algebra with a view Toward Algebraic Geometry. Springer.

• R. Harshorne, Algebraic Geometry, Springer, ISBN 0387902449.

• E. Kunz, Introduction to Commutative Algebra and Algebraic Geometry, Birkhauser, 1985 (1984?). ISBN 0817630651,

• H. Matsumura, *Commutative Ring Theory*, Cambridge Stud. in Adv. Math. 8, (1986,1990,1992). ISBN 0521367646.

• J.-P. Serre, Local Algebra, Springer Monographs in Math. (2000). ISBN 3540666419.

Course Plan:

We will cover commutative algebra along with some related topics in affine algebraic geometry. We will cover as much as possible from the following list of topics:

1. Basic notions: ring homomorphisms, ideals, modules, fundamental theorems

- 2. Prime and maximal ideals; the nilradical and Jacobson radical; Nakayama's Lemma
- 3. $\operatorname{Spec}(A)$; the Zariski topology
- 4. Localization; local properties such as flatness
- 5. Schemes: Affine schemes; Sheaves, locally ringed spaces, and schemes
- 6. Integral extensions: Integral dependence; The going-up and going-down theorems; Normality

7. Noether Normalization and its consequences: Hilbert's Nullstellensatz, geometric meaning thereof; Transcendental dimension

8. Noetherian rings: Hilbert Basis Theorem; Discrete valuation rings

9. Completions: Artin-Rees lemma; For Noetherian rings and modules; Associated graded rings

10. Dimension Theory: For Noetherian local rings; Krull's principal ideal theorem; Regular local rings; Normal irreducible curves are smooth; Normality in codimension 1

11. Formally étale and smooth maps; Jacobian criterion; Cohen-Macauley modules and complete intersections

12. Affine Group schemes: applications to, and proof that they are smooth in characteristic zero.

Grading policy:

To get an "A" in this course, you must

1) Attend the lectures;

2) Make a solid effort on the homework assignments.

The homework assignments will be relatively infrequent (I have not yet decided how many there will be, or exactly how I will assign grades...)).

There will be no in-class exams of any kind.

A grade of less than "A" will result if you fail to attend the course regularly and/or make a less than serious effort to do the homework

If you must miss class for an extended period or cannot return a homework assignment, contact me if you still want an "A" (if truly necessary, I will arrange to give you a ten-minute oral exam to convince me you have learned something about the missed material, thus salvaging the desired "A").

The homework will appear on the course web-site (see http://www.math.umd.edu/~tjh). You should consult that web-site regularly for other announcements related to this course as well.