Math 111, section 6.1 Sets and Set Operations

notes by Tim Pilachowski

Our first topic is one which will (hopefully) clarify the ideas underlying *probability*.

Definition: A **set** is

The individual objects in a set are called

Every set has three properties.

1.

2.

3.

Sets are usually represented by capital letters, such as *A*, *B*, *C*, etc. Notationally, a set is indicated using braces (squiggly brackets). The elements of a set can be defined as a descriptive sentence, list, or equation.

G = the set containing the letters "x", "ö", "A", and the integers "0", "9", "12" = {0, x, ö, 9, 12, A}

H = the set of colors of lights in a standard traffic signal =

I = the set of "solutions to the equation $x^2 = 4$" = $\{x | x^2 = 4\}$ read "the set of elements *x* such that $x^2 = 4$".
This version is called **set-builder notation**.

So, as a list, *I* =

J = the set of positive even numbers =

The symbol $\in$ means "is an element of".

Examples A: Let *G*, *H*, *I*, and *J* be as defined above.

The symbol $\notin$ means "is not an element of":

Examples A (continued): Let *G*, *H*, *I*, and *J* be as defined above.

Two sets A and B are called **equal,** i.e. $A = B$, when they have exactly the same elements. When a set is defined by listing its elements the list may be in any order.

Examples B: Let H = {red, yellow, green}

If a set has no members, it is called the **empty set** or the **null set,** and is denoted either by empty braces, { }, or by the symbol $\varnothing$.

Important note:

One set A is a **subset** of another set B if every element found in set A is also in set B. Another way to say this is that there is nothing found in A which is not also found in B.

Examples C: Let $I = \{x \mid x^2 = 4\} = \{-2, 2\}$, $J = \{2, 4, 6, 8, \ldots\}$, and $K = \{4, 8, 12\}$.

Note the difference between a "subset" and a "proper subset", in both concept and notation.

Theorems about subsets:

Examples C revisited: Let $I = \{x \mid x^2 = 4\} = \{-2, 2\}$, $J = \{2, 4, 6, 8, \ldots\}$, and $K = \{4, 8, 12\}$. List all of the subsets of I. List all of the subsets of K. (Note that set J would have an infinite number of subsets.)

We'll be considering three fundamental set operations.

From two given sets A and B we can make a new set that consists of all the elements of A and all the elements of B. This new set is called the **union** of A and B and is represented by the symbol $A \cup B$. (The union symbol is *not* the letter U.)

Example D: Let Q = {a, b, c, d} and let R = {c, d, e}.

The union of two sets is defined in symbols as follows: $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$. Note that this is a *non-exclusive* use of the word "or": the elements can be in A, or in B, or possibly in both A and B.

Other notes:

From two given sets A and B we can make a new set that consists of all the elements that belong to both A and B at the same time. This new set is called the **intersection** of A and B and is represented by the symbol $A \cap B$.

Example D (continued): Let Q = {a, b, c, d} and let R = {c, d, e}.

The intersection of two sets is defined in symbols as follows: $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$.

Other notes:

Two sets whose intersection is empty are called **disjoint**, i.e. two sets A and B are disjoint if and only if $A \cap B = \varnothing$.

Before addressing the operation of **complement**, it is necessary to define a **universal set**, containing all the individual objects under consideration. For example, if the sets being studied consist of men, women, boys, and girls in a population, then the universal set is everyone in the population. In a primary school mathematics classroom, the universal set contains only positive rational numbers. In a typical algebra classroom, the universal set contains all real numbers, positive and negative, rational and irrational. The letter U (***not*** the union symbol $\cup$) is used to denote the universal set for a given situation.

The complement of a set A is the set of all elements in the universal set that are not members of A, and is represented by the symbol A^c. It is defined in symbols as follows: $A^c = \{x \mid x \in U \text{ and } x \notin A\}$.

Example E. Let U = the set of positive whole numbers = {1, 2, 3, 4, … } and J = the set of positive even numbers = {2, 4, 6, 8, …}.

Complement literally means “that which completes”, and if you combine a set with its complement, you get everything, i.e. the universe.

Other notes:

Venn diagrams provide a visual means of considering sets, even when the particular elements may not be known. In a Venn diagram a rectangle represents the universe set under consideration and circles within the rectangle represent sets within the universe. The operations above would be diagrammed as follows:

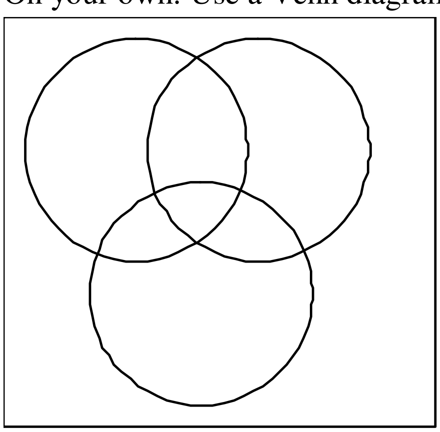
$A \cup B$ | $A \cap B$ | $A \cap B = \varnothing$ (one way)

$A \cap B = \varnothing$ (another way) | A^c | $A \subset B$

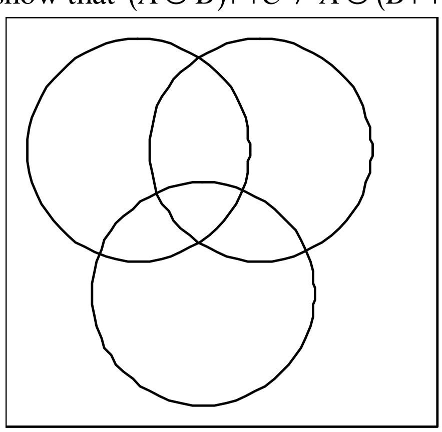
Venn diagrams can also be used with three (or more) sets.

Example F: Draw a Venn diagram to illustrate first $(A \cup B) \cap C$ then $(A \cup B) \cap C^c$.

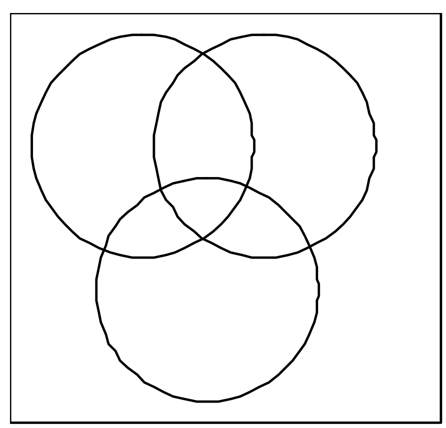
On your own: Use a Venn diagram to show that $(A \cup B) \cap C \neq A \cup (B \cap C)$.



$(A \cup B) \cap C$



$(A \cup B) \cap C^c$



$A \cup (B \cap C)$

Your text introduces the distributive laws for unions and intersections

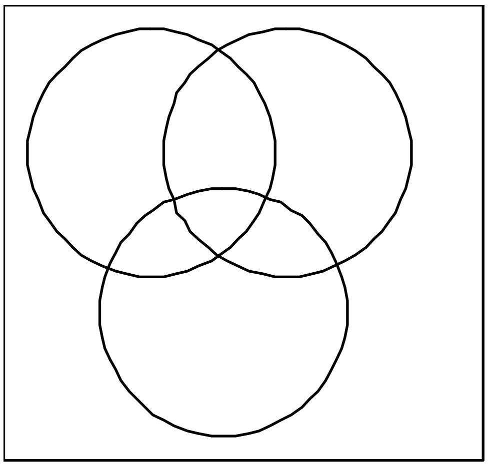
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \quad \text{and} \quad A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

as well as De Morgan's Laws

$$(A \cup B)^c = A^c \cap B^c \quad \text{and} \quad (A \cap B)^c = A^c \cup B^c$$

but none of these will be useful enough in this course for you to worry about memorizing them. Proving them for yourself, using Venn diagrams, can be a good exercise.

Examples G: Consider the universe U = {a, b, c, d, e, f, g, h, i, j, k}
and sets M = {a, b, c, d}, N = {b, c, d, e, f, g}, and P = {g, h, i}.



$M^c =$

$M \cup N =$

$M \cap N =$

$M \cup P =$

$M \cap P =$

$N \cup P^c =$

$N \cap P^c =$

$M \cap (N \cup P^c) =$

$M \cup (N \cap P^c) =$

Now let T = {c, e, b, d}. What can we say about T in relationship to M, N, and $M \cap N$?

Example H: Let U = the members of a freshman class at UMCP, A = the set of students who have academic scholarships, B = the set of students who are athletes, and C = the set of students who live on campus.

Describe each set below [parts a) through e)] in words.

a) A^c

b) $B \cap C$

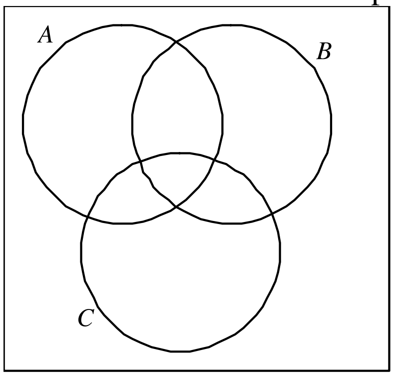
c) $A \cap B^c$

d) $A \cup B$

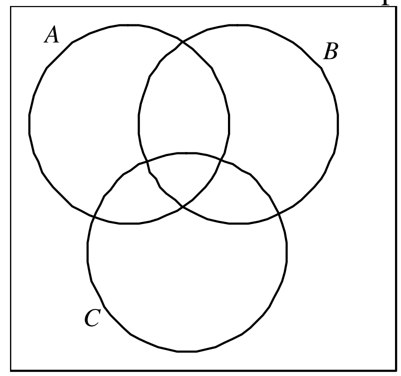
e) $B \cup C^c$

Write the set that represents (symbolically) each statement f) through k) below, then also draw a Venn diagram to illustrate each set.

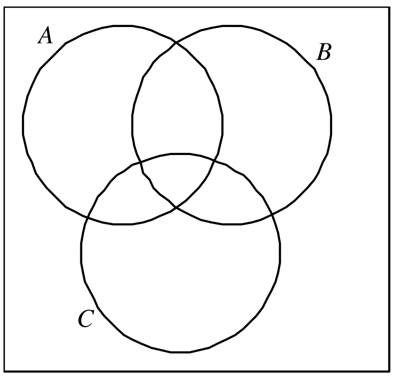
f) the set of students who do not live on campus



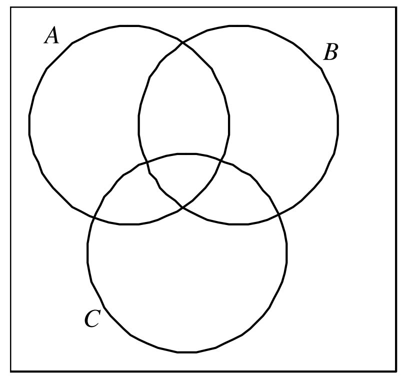
g) the set of athletes who have academic scholarships



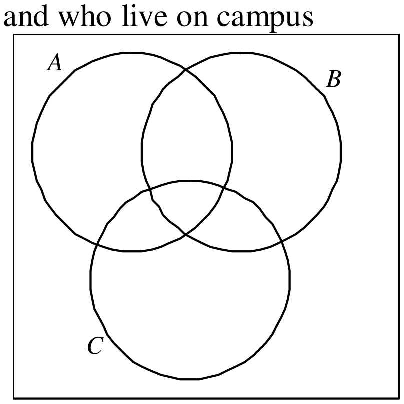
h) the set of athletes who don't live on campus



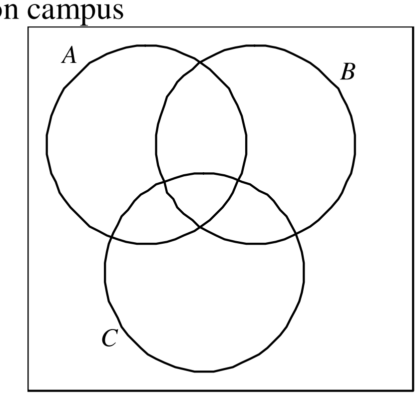
i) the set of students who have academic scholarships or live on campus.



j) the set of athletes who have academic scholarships and who live on campus



k) the set of athletes who have academic scholarships who do not live on campus



Some closing notes: The symbols $\cup$ and $\cap$ for union and intersection are pretty much standard; other symbology is not. While some texts in set theory use our notation A^c for complement, others use $\overline{A}$ or A'.

Important note: The ideas presented here about sets have parallels in the rest of the work done in the rest of the class on probability.