Math 111, section 6.2 The Number of Elements in a Set

notes by Tim Pilachowski

Definition: A **set** is a well-defined collection of objects.

The individual objects in a set are called the **elements** or **members** of the set.

The number of elements in a set S (sometimes called the cardinality of the set) is designated by $n(S)$.

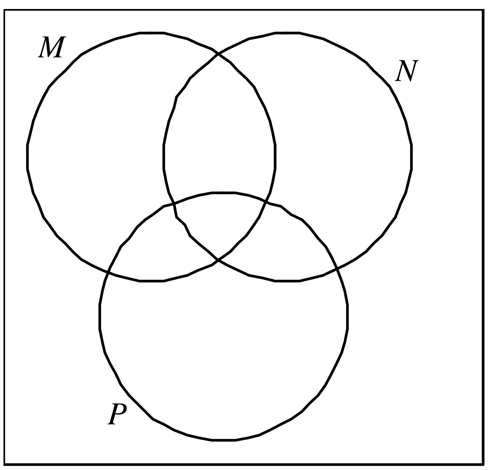
Examples A (6.1 Examples A revisited): Let G = the set containing the letters "x", "ö", "A", and the integers "0", "9", "12" = {0, x, ö, 9, 12, A}, H = the set of colors of lights in a standard traffic signal = {red, yellow, green}, $I = \{x \mid x^2 = 4\}$, J = the set of positive even numbers = {2, 4, 6, 8, …}.

$n(G) =$ $\quad$ $n(H) =$

$n(I) =$ $\quad$ $n(J) =$

$n(\varnothing) =$ $\quad$ $n(\{\varnothing\}) =$

Examples B (6.1 Examples G revisited): Consider the universe U = {a, b, c, d, e, f, g, h, i, j, k} and sets M = {a, b, c, d}, N = {b, c, d, e, f, g}, and P = {g, h, i}.



$n(M) =$ $\quad$ $n(N) =$ $\quad$ $n(P) =$

$M^c =$ $\quad$ $n(M^c) =$

$M \cup N =$ $\quad$ $n(M \cup N) =$

$M \cap N =$ $\quad$ $n(M \cap N) =$

$M \cup P =$ $\quad$ $n(M \cup P) =$

$M \cap P =$ $\quad$ $n(M \cap P) =$

$P^c =$ $\quad$ $n(P^c) =$

$N \cup P^c =$ $\quad$ $n(N \cup P^c) =$

$N \cap P^c =$ $\quad$ $n(N \cap P^c) =$

When considering the number of elements in a union, one must do more than add the individual cardinalities. In the example above, the number of elements in $M \cup N$ is less than the sum of the cardinalities of M and N. Note that the elements b, c and d are included when counting the number of elements in M, then again when counting the number of elements in N. But since the elements b, c and d are only included once each in the union, we must find a way to discount the duplication when counting the cardinality of the union.

The mathematical relationship for the cardinality of the union of any two sets A and B is given in the formula

$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$

This is an important formula. Remember it! We'll need it for probability.

In terms of the example above:

$n(M \cup N) =$

and $n(N \cup P^c) =$

The case of disjoint sets is just a particular application of the formula above:

$n(M \cup P) =$

If we know any three of the cardinalities in the formula $n(A \cup B) = n(A) + n(B) - n(A \cap B)$, we can find the fourth.

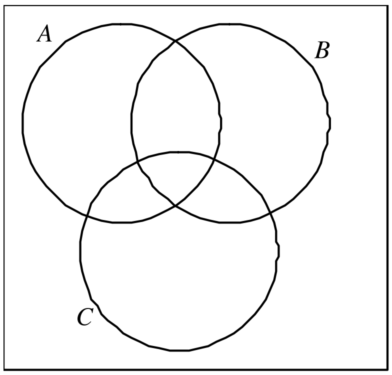
Examples C:

a. $n(A) = 20$, $n(A \cap B) = 6$, $n(A \cup B) = 30$, $n(B) = ?$

b. $n(C) = 18$, $n(D) = 15$, $n(C \cup D) = 40$, $n(C \cap D) = ?$

Example D. is on the next page.

Example D: Among the 178 members of a freshman class at Matriarch University (U. Mama), 37 have academic scholarships, 55 are athletes, 62 live on campus, 14 are athletes who have an academic scholarship, 15 have an academic scholarship and live on campus, 21 have are athletes who live on campus, and 3 are athletes with academic scholarships who live on campus. Use a Venn diagram to answer the following questions: a) How many students are athletes who do not have an academic scholarship and who live off-campus? b) How many students are athletes with an academic scholarship, but who do not live on campus? c) How many students fit in none of the three categories?
answers: a) 23, b) 11, c) 71



Let U = the members of a freshman class at U. Mama, A = the set of students who have academic scholarships, B = the set of students who are athletes, and C = the set of students who live on campus.

a) How many students are athletes who do not have an academic scholarship and who live off-campus?

b) How many students are athletes with an academic scholarship, but who do not live on campus?

c) How many students fit in none of the three categories?

Side note: Our text uses $n(S)$ for the number of elements in a set. Others use $c(S)$ [for cardinality of S] or $|S|$.