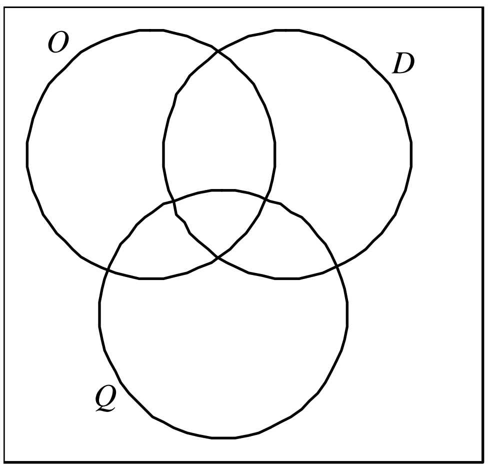
Math 111, section 6.3 The Multiplication Principle

notes by Tim Pilachowski, Fall 2011

6.2 Example E: While he was a student at Whatsamatta U. in Frostbite Falls, Bullwinkle Moose was a star on the football team. On the team roster, there were 4 players who could be quarterback, 19 who could play offense (excluding the quarterback position), and 18 who could play defense. Among these players there were 10 who could play either defense or non-quarterback offense, 3 quarterbacks who could play another offensive position, and 1 defensive player who could switch to being quarterback in a pinch. Only Bullwinkle could play defense, offense and quarterback. Also, there are 3 special teams players, who are neither offense, nor defense, nor quarterback. a) How many players are on the roster? b) How many team members play only quarterback? c) One region in the Venn diagram has a cardinality of 0. Describe (first verbally then using set notation) the players who would fall into this region, were any to be there. d) How many players are not on the defensive roster? e) How many players do not play any offensive position, including not being quarterback? f) How many players can fill exactly two types of positions? g) How many players can fill at least two types of positions? h) How many players can fill at most two types of positions?

answers: a) 31, b) 1, c) … , d) 13, e) 11, f) 11, g) 12, h) 30



In section 6.3 we come to counting that's even more complicated. We won't be counting just elements in sets and subsets, but rather the number of possibilities when we're looking at a series of events. We have to ask the question, "What might happen?" "What are the possible outcomes?" "How many different ways are there?"

Examples A: a) Mario tosses a dime onto a table. How many possible outcomes are there? b) How many results are possible when Luigi tosses a dime and a nickel? c) If Daisy rolls a standard six-sided die, how many things could happen? d) How many distinct events might there be if Peach rolls one red die and one green die? e) Yoshi, trying to be different, tosses one coin and rolls one die. How many outcomes are possible?

Like the elements in a set, the possibilities in Example A are distinct; there are no in-betweens. Later on, the technical word we'll introduce is "discrete", which essentially means that the possibilities/outcomes/events are countable.

The Multiplication Principle:

Examples A continued: f) Toad tosses a dime, a nickel and a penny onto a table. How many possible outcomes are there? g) If Toadette rolls three standard six-sided dice, how many results could there be? h) How many different outcomes might there be if Bowser tosses three coins and rolls three dice?

Example B: In a poker game, using a standard deck of fifty-two playing cards, a royal straight is A-K-Q-J-10. a) How many ways are there to get a royal straight? b) A royal straight flush is A-K-Q-J-10 where all five cards are the same suit. How many ways are there to get a royal straight flush?

Example C: There are three major routes from the Baltimore Beltway to the Washington DC Capitol Beltway: I95, US 1, and the BW Parkway. From Washington to Annapolis there are two primary routes: US 50 and Md 450. How many ways can Thaddeus' GPS get him from Baltimore to Annapolis via Washington (assuming the software keeps him on major routes)?

Example D: A club is electing its three top officers. There are three candidates for President, two candidates for Vice President, and four for Secretary/Treasurer. Find the number of ways the election can turn out.

Example E: Farnsworth is shopping for a new laptop computer. Mega-Biggie Comp-U-World offers customers 3 screen sizes, 4 CPU speeds, and 6 sizes of RAM. Farnsworth can also choose to include (or not) a built-in webcam. How many different (distinct) ways can he have his computer built?

Example F: The UMCP Math Placement Test has 70 multiple-choice questions. Each question presents a student with six choices: a, b, c, d, e, and "leave it blank". How many different ways are there of answering the questions on the UMCP Math Placement Test?

Example G: Denny's restaurants advertise their 2-4-6-8 value menu, stating "the possibilities are endless". a) In what way are the possibilities endless? b) The value menu has six \$2 items, four \$4 items, four \$6 items, and four \$8 items. If Esperanto has decided to order one item of each price, how many distinct ways can she order her meal?

Example H: During Baltimore Restaurant Week in August 2011, Chiapparelli's Restaurant offered a three-course lunch for \$20.11 and a three-course dinner for \$35.11. The limited menu listed the follow choices:

Lunch

First Course (choice of one)
- Old School Maryland Crab Soup
- Fried Calamari
- Prosciutto and Seasonal Fruit
- Chiapparelli's Famous House Salad

Second Course (choice of one)
- Grandma's Ravioli
- Grilled Chicken Pesto Panini
- Maryland Jumbo Lump Crab Cake
- Eggplant Parmigiana

Third Course (choice of one)
- Mini Cannoli

Dinner

First Course (choice of one)
- Stacked Eggplant with local Goat Cheese
- Italian Style Moules Frites
- Fried Calamari
- Scallops alla Cole

Second Course (choice of one)
- Seafood Ravioli
- Roasted Veal
- Vegetarian Spaghetti Carbonara
- Paella
- Chicken Parmigian
- Maryland Jumbo Lump Crab Cake

Third Course (choice of one)
- Cannoli
- Homemade Tiramisu

How many diners would be required to have every possible combination served for lunch? For dinner?

Example I: Maryland license plates for passenger cars (as opposed to trucks and commercial vehicles) always used to be three letters followed by three numbers. a) How many license plates were possible using this schema? b) Two dozen specific in-order three-letter arrangements were disallowed because they suggested offensive words. (All number arrangements were permitted.) How many possible license plates were disallowed? c) How many possible license plates were there with the given restriction? d) When "three letters followed by three numbers" had all been used, Maryland began using "all six could be either letters or numbers". How many new arrangements were there to choose from (when compared to the scenario in a)?

Note that in the scenario of Example I, letters and numbers can be repeated. Later on we'll look at the case where duplications are either not allowed or not possible.

Example J: Back in lecture 6.1 I stated that the number of subsets of a given set is 2^n, where n is the cardinality of the set. Can we prove this is true using the multiplication principle?