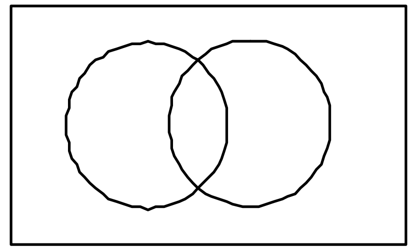
Calculus 111, section 7.3 Rules of Probability

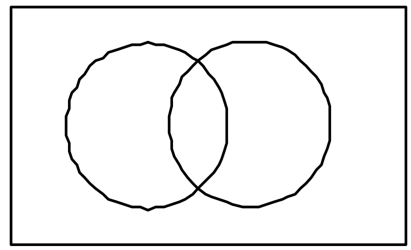
notes by Tim Pilachowski

We have defined our sample spaces and events using the terminology of sets. So far, in the examples we've used, it has been possible to list all of the simple events in a sample space. For larger sample spaces, this may be overwhelming, or maybe not even possible.

We are going to borrow something else from set theory: Venn diagrams. Venn diagrams can be used to keep track of either the simple events in a sample space *or* their associated probabilities.

Example A: For a sample space $S = \{ s_1, s_2, s_3, s_4, s_5, s_6 \}$ all simple events are equally likely. Let $A = \{ s_1, s_2, s_3, s_4 \}$ and let $B = \{ s_3, s_4, s_5 \}$.





$P(A) =$ $\qquad$ $P(B) =$

$A \cap B =$ $\qquad$ $P(A \cap B) =$

$n(A) =$ $\qquad$ $n(B) =$

$n(A \cap B) =$ $\qquad$ $n(A \cup B) =$

$P(A \cup B) =$

Let $C = \{ s_6 \}$.

$A \cap C =$ $\qquad$ $P(A \cap C) =$

$B \cap C =$ $\qquad$ $P(B \cap C) =$

$P(A \cup C) =$

$P(B \cup C) =$

$A^c =$ $\qquad$ $P(A^c) =$

Example B: You toss two standard six-sided dice, one red and one white.

$S = \{$	(1, 1),	(2, 1),	(3, 1),	(4, 1),	(5, 1),	(6, 1),
	(1, 2),	(2, 2),	(3, 2),	(4, 2),	(5, 2),	(6, 2),
	(1, 3),	(2, 3),	(3, 3),	(4, 3),	(5, 3),	(6, 3),
	(1, 4),	(2, 4),	(3, 4),	(4, 4),	(5, 4),	(6, 4),
	(1, 5),	(2, 5),	(3, 5),	(4, 5),	(5, 5),	(6, 5),
	(1, 6),	(2, 6),	(3, 6),	(4, 6),	(5, 6),	(6, 6) $\}$

A = the red die is a 1; $P(A)$ =

B = the white die is a 1; $P(B)$ =

$A \cap B =$ $\quad$ $P(A \cap B) =$

$A \cup B =$ $\quad$ $P(A \cup B) =$

$A^c =$ $\quad$ $P(A^c) =$

$(A \cup B)^c =$ $\quad$ $P\left[(A \cup B)^c\right] =$

Example C: You pick a card from a standard deck of 52 cards. [4 suits: spades (S), hearts (H), diamonds (D), clubs (C); 13 cards in each suit: ace (A), king (K), queen (Q), jack (J), 10, 9, 8, 7, 6, 5, 4, 3, 2]

A = the card is an Ace; $P(A)$ =

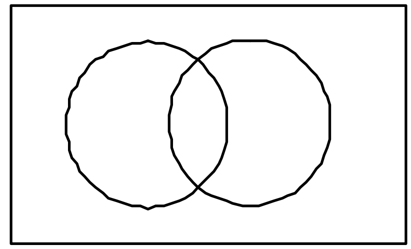
B = the card is a Spade; $P(B)$ =

C = the card is the Ace of Spades; $P(C)$ =

D = the card is an Ace or a Spade; $P(D)$ =

E = the card is neither an Ace nor a Spade; $P(E)$ =

Example D: Silver Springs, Florida, has a snack bar and a gift shop. The management observes 100 visitors, and counts 65 who eat in the snack bar (F), 55 who make a purchase in the gift shop, and 40 who do both. a) What is the probability that a visitor will not buy anything in the gift shop? b) What is the probability that a visitor will either eat in the snack bar or buy something in the gift shop? c) What is the probability that a visitor will buy something in the gift shop but not eat in the snack bar?



Example E: In 2009, households were surveyed about health insurance coverage, with the following results.

	Age 18-64	Age < 18	Totals
Public Plan	10286	10771	21057
Private Plan	47000	15914	62914
Uninsured	14144	1885	16029
Totals	71430	28570	100000

DATA SOURCE: CDC/NCHS, National Health Interview Survey, 1997-2009, Family Core component. Data are based on household interviews of a sample of the civilian noninstitutionalized population. Numbers given above were extrapolated from summary tables.

E_1 = individual is under 18; $P(E_1)$ =

E_2 = individual has health insurance; $P(E_2)$ =

E_3 = individual is under 18 ***and*** has health insurance; $P(E_3)$ =

E_4 = individual is under 18 ***or*** has health insurance; $P(E_4)$ =

Example F: Among the 178 members of a freshman class at Matriarch University (U. Mama), 37 have academic scholarships, 55 are athletes, 62 live on campus, 14 are athletes who have an academic scholarship, 15 have an academic scholarship and live on campus, 21 are athletes who live on campus, and 3 are athletes with academic scholarships who live on campus.

Remember this one from Lecture 6.2? You have something similar in the assigned textbook practice exercises. In 6.2 the questions were about numbers in various sets and subsets. Now, in section 7.3 the questions are about probabilities.

I suggest you use the same approach as in 6.2 – draw and fill in a Venn diagram, then once all of the numbers are in place turn them into the probabilities you need by dividing by $n(U)$.