

Calculus 111, section 7.4 Counting Techniques and Probability

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In chapter 6, the question was, “How many ways...”, and we counted the number of possible outcomes. In section 7.1, we constructed sample spaces by asking, “What could happen?” In sections 7.2 and 7.3 we defined probability and imported the addition (union-intersection) and complement formulae. Now, in section 7.4, we bring in counting techniques from chapter 6: permutations and combinations.

For simple events that are equally likely to occur, we can use a “uniform probability model”. Formally, for an event E ,

$$P(E) = \frac{\text{number of ways } E \text{ can happen}}{\text{number of possible outcomes}} = \frac{\text{number of simple events in } E}{\text{number of simple events in } S} = \frac{n(E)}{n(S)}.$$

We also have the addition principle, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, and the complement principle, $P(A^c) = P(S) - P(A) = 1 - P(A)$.

Example A: You toss two coins. The sample space is $S = \{ HH, HT, TH, TT \}$.

Notes on probabilities and the sample space:

$A =$ both coins are heads $= \{ HH \}$

$B =$ exactly one coin is heads $= \{ HT, TH \}$

$C =$ at least one coin is heads $=$

We could have gotten the same results by thinking from a different perspective.

For $S =$ two coins are tossed,

For $A =$ both coins are heads

For $B =$ exactly one coin is heads

$P(A) =$

$P(B) =$

$P(C) =$

Example A extended: You toss ten coins and record the results.

We could list heads-tails outcomes:

$S = \{ \text{HHHHHHHHHH, THHHHHHHHH, HTHHHHHHHH, HHTHHHHHHH, HHHTHHHHHH, ...}$
 $\text{TTTTTTTTTT} \}$, but we're not going to. We don't have to. All we need to know is that, by the multiplication principle we have "2 choices for the 1st coin times 2 choices for the 2nd times...". That is,

$$n(S) =$$

$A =$ exactly three coins are heads

$B =$ no more than three coins are heads

$C =$ at least four coins are heads

Homework exercises from the text which ask about coins would follow the same reasoning as in the Example above. So would exercises about boys and girls in families, and true-false questions on tests. In both cases, there are two choices for each "slot" to be filled in.

Example B: Because we can be omniscient, we know that there are 25 defective spark plugs in a production run of 1000. Quality control workers (who are not omniscient) pick ten spark plugs to test.

$$n(S) =$$

a) What is the probability that all ten are defective?

b) What is the probability that at least two are defective?

Example C: You pick four cards from a standard deck of 52.

$$n(S) =$$

a) What is the probability of picking the four Aces?

b) What is the probability that two of the four cards are Aces?

c) What is the probability of picking four of a kind?

Example D: A box contains 3 blue blocks and 2 yellow blocks. You pick two blocks.

$n(S) =$

a) What is the probability of picking two blue blocks?

b) What is the probability of picking one blue and one yellow block?

c) What is the probability of picking two yellow blocks?

Example E: A club has twenty members, 12 female and 8 male. The club is selecting its Solstice Dance committee by pulling six names from a hat.

$n(S) =$

a) What is the probability that Dickens is selected?

b) What is the probability that both Dickens and his bffn Dickenson are selected?

c) If the committee must have three male and three female members, what is the probability that both Dickens (male) and his bffn Dickenson (female) are selected?