111, section 8.3 Variance and Standard Deviation

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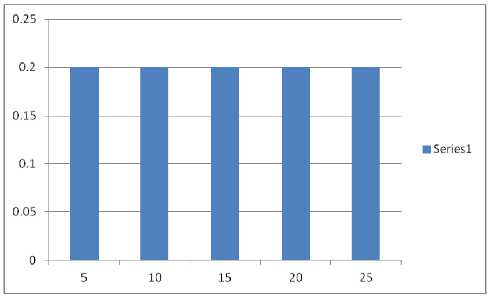
An expected value (mean, average) gives us what is called a "measure of central tendency", an idea of where the "middle" lies. However, a mean alone is insufficient for providing a good idea of the distribution of the data.

Examples A: The four histograms below represent four sets of data. a) Show that the expected value/mean equals 15 for each of them. b) We'll come back to this: variance and standard deviation.

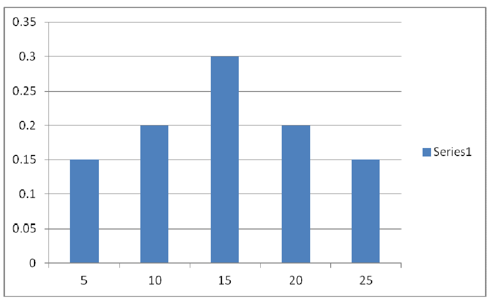


x	5	10	15	20	25
$P(X = x)$					

$E(X) = \mu =$

Var $(X) =$

$\sigma =$

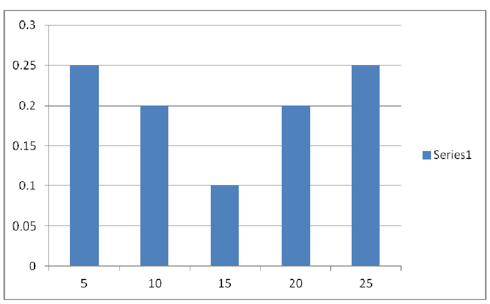


x	5	10	15	20	25
$P(X = x)$					

$E(X) = \mu =$

Var $(X) =$

$\sigma =$

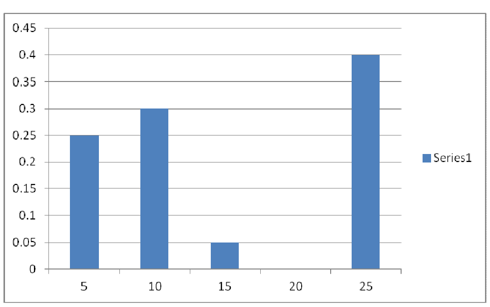


x	5	10	15	20	25
$P(X = x)$					

$E(X) = \mu =$

Var $(X) =$

$\sigma =$



x	5	10	15	20	25
$P(X = x)$					

$E(X) = \mu =$

Var $(X) =$

$\sigma =$

Enter two new, but related concepts: *variance* and *standard deviation*.
For a probability distribution with expected value $E(X) = \mu$,

$$\operatorname{Var}(X) = \sigma^2 = p_1 * (x_1 - \mu)^2 + p_2 * (x_2 - \mu)^2 + \ldots + p_n * (x_n - \mu)^2 = \sum_{i=1}^{n} p_i * (x_i - \mu)^2$$

Variance can be thought of as "sum of [probability times (value minus mean) squared] ".

$$\text{standard deviation of } X = \sigma = \sqrt{\operatorname{Var}(X)}$$

Variance and standard deviation are both measures of how much the amounts (x_i) vary (or deviate) from the mean ($E(X) = \mu$).

Examples A revisited: b) Go back to the four sets of data in Example A and calculate the variance and standard deviation for each of them.

8.1-2 Example F revisited: Suppose we measure the heights of 25 people to the nearest inch.

height (in.)	64	65	66	67	68	69	70
probability	0.2	0.28	0.24	0.16	0.04	0	0.08

Given a mean height of 65.88 in., calculate the variance and standard deviation.

8.1-2 Example A-2 revisited: You toss a coin ten times. For random variable X = number of heads in ten tosses of a coin, and given a mean of 5, calculate the variance and standard deviation.

x	0	1	2	3	4	5	6	7	8	9	10	
$P(X = x)$	$\frac{1}{1024}$	$\frac{5}{512}$	$\frac{45}{1024}$	$\frac{15}{128}$	$\frac{105}{512}$	$\frac{63}{256}$	$\frac{105}{512}$	$\frac{15}{128}$	$\frac{45}{1024}$	$\frac{5}{512}$	$\frac{1}{1024}$	total = 1

8.1-2 Example B revisited: You roll two dice. For random variable X = the sum of the two dice, and given a mean of 5, calculate the variance and standard deviation.

x	2	3	4	5	6	7	8	9	10	11	12	
$P(X = x)$	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$	total = 1

8.1-2 Example C revisited: You deal five cards from a standard deck of 52. For random variable X = number of Aces, and given expected value ≈ 0.31463, calculate the variance and standard deviation.

x	0	1	2	3	4	
$P(X = x) \approx$	0.65884	0.29947	0.03993	0.00174	0.00002	total = 1

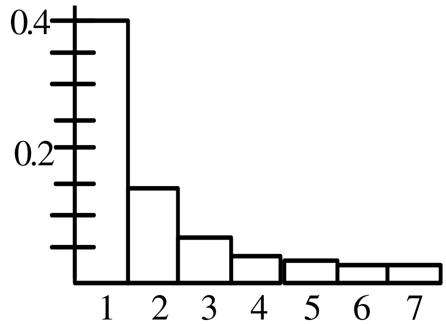
8.1-2 Example D revisited: Suppose that you pick three blocks without replacement from a box that contains 3 blue blocks and 2 yellow blocks. For X = number of blue blocks drawn, and given expected value = 1.8, calculate the variance and standard deviation.

x	1	2	3	
$P(X = x)$	$\frac{3}{10}$	$\frac{3}{5}$	$\frac{1}{10}$	total = 1

8.1-2 Example E revisited. Let X = the number of days each ICU patient stays in intensive care.

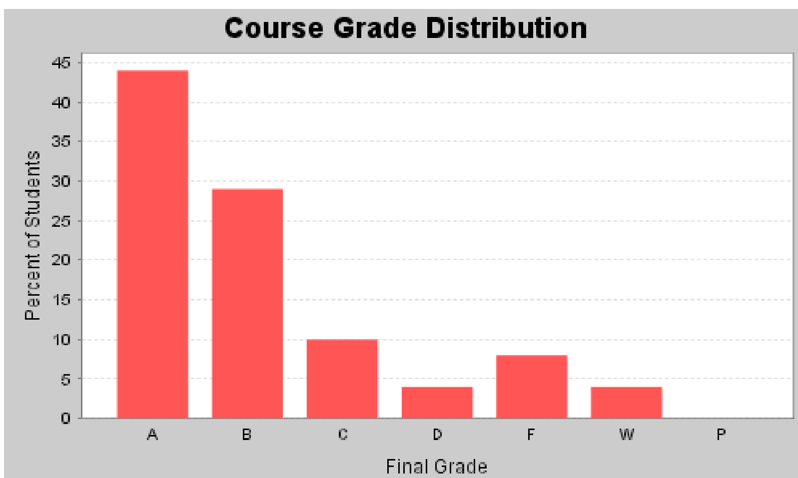
$$X = 1, 2, 3, 4, 5, \ldots$$

The probabilities would be developed based on relative frequencies—observations made from hospital and patient records. The histogram might look something like this.



For your general knowledge, this probability distribution is approximately exponential, with formula $f(x) = 0.4e^{-0.4x}$. Calculus is needed to find the variance and standard deviation.

8.1-2 Example G revisited: A Math 220 class, taught in the Fall of 2010 at UMCP, had the following grade distribution. Define X = grade points (GPs), where an A is 4 GPs, a B is 3 GPs, a C is 2 GPs, a D is 4 GPs, and an F or W is 0 GPs. Given expected value ≈ 2.893, calculate the variance and standard deviation of grade point average for this class.



x	4	3	2	1	0	
$P(X = x)$	0.443	0.293	0.100	0.042	0.122	total = 1

8.2 Example H revisited: Insurance companies use actuarial data to set rates for policies. Collected data indicate that, on a \$1000 policy, an average of 1 in every 100 policy holders will file a \$20,000 claim. An average of 1 in every 200 policy holders will file a \$50,000 claim. An average of 1 in every 500 policy holders will file a \$100,000 claim. Given expected value = 350, calculate the variance and standard deviation for the value of a policy to the company.

X = value of a policy	-19000	-49000	-99000	1000	
$P(X = x)$	0.01	0.005	0.002	0.983	total = 1

8.2 Example J revisited: An airline prices 150 seats according to the following schema. Assuming the aircraft is sold out, and given expected value = 557.80, calculate the variance and standard deviation of the value of a ticket to the airline.

type	first class	unrestricted coach	restricted coach	frequent flyer
number	20	45	81	4
price	\$1200	\$750	\$320	\$0

8.2 Example K revisited: Five coins are tossed. If 0, 1, or 2 heads come up, the player wins nothing. If 3 heads come up the player wins \$2. If 4 heads come up the player wins \$5, and if 5 heads come up the player wins \$10. Given mean ≈ 1.72, calculate the variance and standard deviation for the value of winnings paid out by the house.

no. of heads	0	1	2	3	4	5	
X = winnings	0	0	0	2	5	10	
$P(X = x)$	$\frac{1}{32}$	$\frac{5}{32}$	$\frac{5}{16}$	$\frac{5}{16}$	$\frac{5}{32}$	$\frac{1}{32}$	total = 1