Calculus 130, section 4.2 Derivatives of Products and Quotients

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In section 4.1 we found one formula for finding a derivative: the power rule. Everything else was extensions or applications of the same thing. Now we present two new rules for finding derivatives—these are ones you'll need to memorize.

The first is the *product rule*: $\frac{d}{dx}[u(x) * v(x)] = u(x) * v'(x) + v(x) * u'(x) = uv' + vu'$. The text does a derivation at the beginning of the chapter. I'll leave it to you to look over that work.

Example A: Given $m(x) = (4x^2 - 1)(3 + x^3)$, find m'. *answer*: $m'(x) = 20x^4 - 3x^2 + 24x$.)

method 1: multiplying out

method 2: product rule

Example B: Given $n(x) = (5x^4 - 1)^2$, find $n'(x)$. *answer*: $n' = 200x^7 - 40x^3$

method 1: multiplying out

method 2: product rule

Example C. Given $f(x)=\left(2x^4-x^2+1\right)\left(\frac{3}{x^4}-\frac{1}{x^2}-1\right)$, find $\frac{df}{dx}$.

Multiplying this out would be time-intensive and fraught with possible error. The product rule provides an easier and more reliable method.

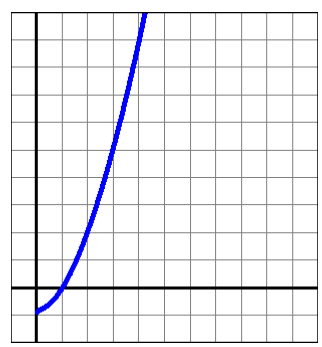
answer: $f'=\left(2x^4-x^2+1\right)\left(-\frac{12}{x^5}+\frac{2}{x^3}\right)+\left(\frac{3}{x^4}-\frac{1}{x^2}-1\right)\left(8x^3-2x\right)$

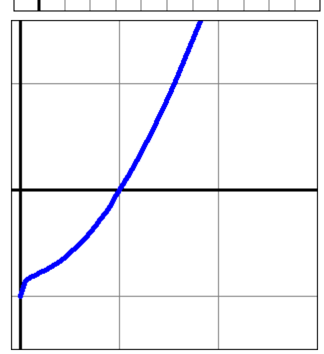
Example D illustrates successive applications of the product rule, with small numbers to be less confusing (hopefully). Example D. Given the polynomial $f(x)=(x+3)\left(x^2+1\right)\left(x^3-1\right)$, find the first derivative.

answer: $f'=(x+3)\left[\left(x^2-1\right)\left(3x^2\right)+\left(x^3-1\right)(2x)\right]+\left(x^2+1\right)\left(x^3-1\right)$

Example E: Given $y=(2x+1)\left(\sqrt{x}-1\right)$ solve $\frac{dy}{dx}=0$.

answer: no solution





Now we move to the *quotient rule*.

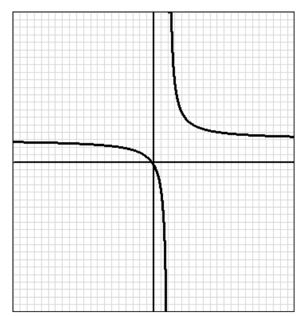
$$\frac{d}{dx}\left[\frac{u(x)}{v(x)}\right]=\frac{v(x)*u'(x)-u(x)*v'(x)}{[v(x)]^2}=\frac{vu'-uv'}{v^2}$$

Example F: Given $f(x)=\frac{\sqrt[3]{x}}{x^2}$, find $f'(x)$. *answer*: $f'(x)=-\frac{5}{3x^{8/3}}$

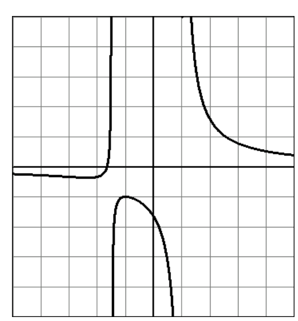
method 1: simplifying first

method 2: quotient rule

Example G: Given $h(x)=\frac{3x+1}{x-2}$ find h'. *answer*: $h'=\frac{-7}{(x-2)^2}$



Example H: Find the two x-values where $f(x)=\frac{3x+5}{2x^2+x-3}$ has a horizontal tangent. *answers*: $x=-\frac{7}{3},-1$



Example I: Given the rational function $f(x)=\frac{(x+3)(x^2+1)}{(x^3-1)}$, find $\frac{d}{dx}[f(x)]$.

answer: $f'=\frac{(x^3-1)*[(x+3)(2x)+(x^2+1)]-[(x+3)(x^2-1)]*(3x^2)}{(x^3-1)^2}$

Examples J: Given the functions $y=\frac{x^5}{5}$, $y=\frac{5}{x^5}$ and $y=\frac{1}{5x^5}$ find the three first derivatives.

answers: $x^4, -\frac{25}{x^6}, -\frac{1}{x^6}$

Example K: Average cost (*AC*) can be thought about as total cost divided by the number of items produced, or $AC=\frac{C(x)}{x}$. A small lampshade manufacturer has determined that the cost to produce *x* lampshades is $C(x)=\frac{1}{8}x^2+4x+200$. Use the quotient rule to determine the *x*-value for which $AC'=0$.

Answer: $x = 40$ *If we don't get to this one during Lecture, do it yourself for practice using the quotient rule.*