Calculus 130, section 5.1-5.2 Functions: Increasing, Decreasing, Extrema

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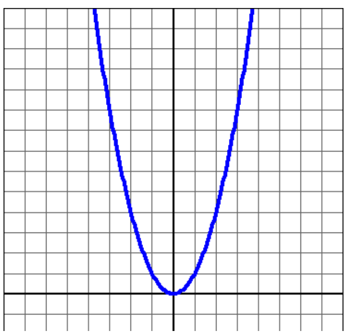
Reminder: You will not be able to use a graphing calculator on tests!
First, a quick scan of what we know so far.

The slope of a curve at a point
= slope of line tangent to the curve at that point
= (instantaneous) rate of change of the curve at that point
= first derivative evaluated at that point: $f'(x)$, $\frac{dy}{dx}$ and D_x.

To find the first derivative of a given function we have
the power rule, constant multiple rule, sum rule,
product rule, quotient rule and chain rule.

Chapter 5 applies all this in various circumstances, and explores the meaning of it all.



Example A. Consider the graph of $y = x^2$ pictured to the left. Reading from left to right—

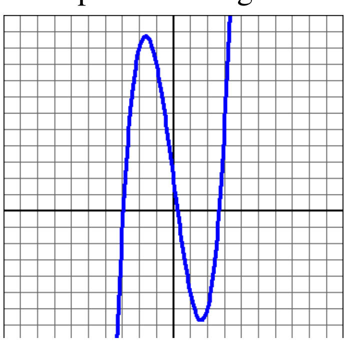
From "forever left" $(-\infty)$ to $x = 0$, the curve is going down = graph is falling = the slope of the curve is negative.

The graph "bottoms out" at the vertex (0, 0) where the slope of the curve = 0.

From $x = 0$ onward to "forever right" (∞) the curve is going up = the graph is rising = the slope of the curve is positive.

In technical terms, the function $f(x) = x^2$ is *decreasing* on the interval $-\infty < x < 0$, has a *minimum* at (0, 0), and is *increasing* on the interval $0 < x < \infty$. We can say that the *minimum value of f* is equal to 0 because there are no lower values in the range of *f*. In other words, the minimum here is an *absolute minimum*. The value $x = 0$ is a *critical number* because the graph has a horizontal tangent and therefore $f'(0) = 0$. The point (0, 0) is called a *critical point*.

Example B: Let's go back to $f(x) = x^3 - 8x + 2$ and take a closer look at the curve, pictured to the left below.



first derivative:

critical numbers:

critical points:

interval(s) increasing:

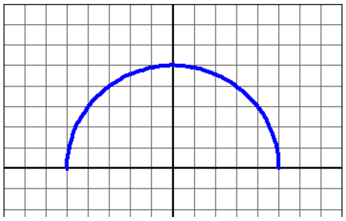
interval(s) decreasing:

relative maximum:

relative minimum:

The function $f(x) = x^3 - 8x + 2$ has no absolute maximum or minimum; the range is $-\infty < x < \infty$. Vocabulary to know: *relative extrema* (plural) and *relative extremum* (singular).

Example C: The function $f(x)=\sqrt{25-x^2}$ has a limited domain, $-5 \le x \le 5$, and range, $0 \le y \le 5$.



first derivative:

critical numbers:

critical points:

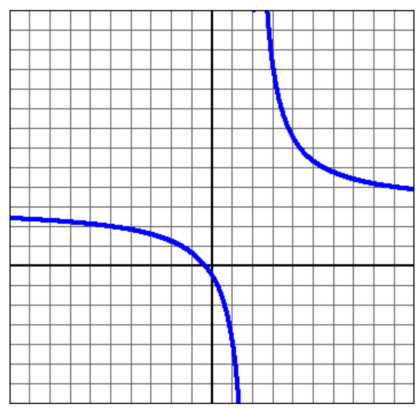
interval(s) increasing:

interval(s) decreasing:

extrema (maximum or minimum):

The maximum value of the function is 5. The minimum value of the function is 0. Because the minimum occurs at the endpoints of the domain it is called an *endpoint extreme value* or *endpoint extremum*.

Example D: Consider the function $f(x)=\dfrac{3x+1}{x-2}$.



first derivative:

critical numbers:

critical points:

interval(s) increasing:

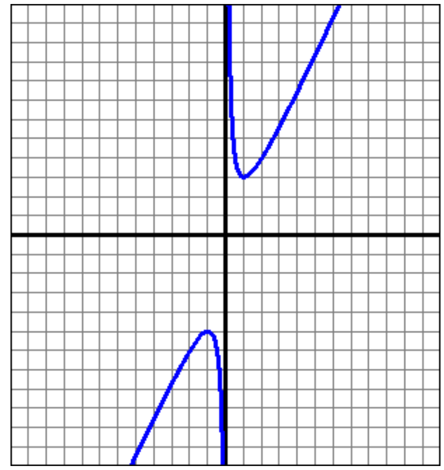
interval(s) decreasing:

extrema (maximum or minimum):

vertical asymptote:

horizontal asymptote:

Example E: Consider the function $f(x) = 2x + \frac{2}{x} - 1 = 2x + 2x^{-1} - 1$.



first derivative:

critical numbers:

critical points:

interval(s) increasing:

interval(s) decreasing:

extrema (maximum or minimum):

vertical asymptote:

horizontal asymptote:

Example F: Consider the function $f(x) = \frac{x^3}{e^x}$.

first derivative:

critical numbers:

critical points:

interval(s) increasing:

interval(s) decreasing:

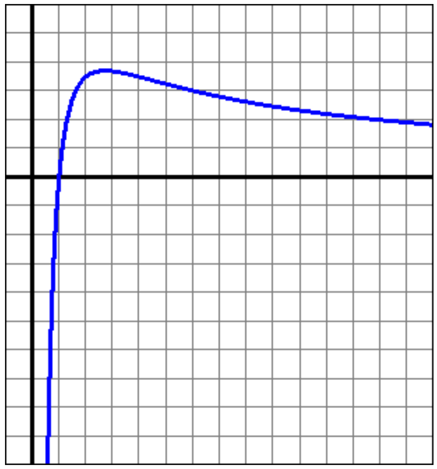
extrema (maximum or minimum):

vertical asymptote:

horizontal asymptote:

Example G: The concentration of a drug in the bloodstream t hours after injection into a muscle is given by $c(t) = 9\left(e^{-0.3t} - e^{-3t}\right)$ units. Find the time at which the concentration of the drug in the bloodstream is at its maximum.

Example H: Consider the function $f(x) = \frac{10 \ln x}{x}$.



first derivative:

critical numbers:

critical points:

interval(s) increasing:

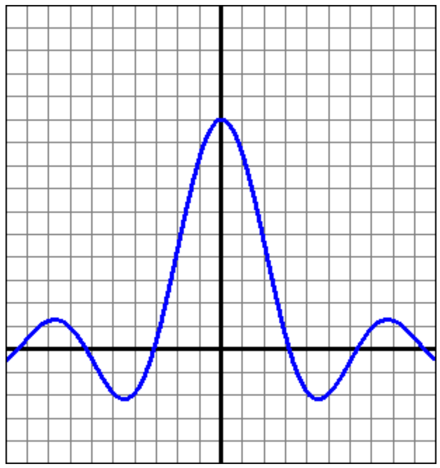
interval(s) decreasing:

extrema (maximum or minimum):

vertical asymptote:

horizontal asymptote:

Example I: Consider the function $f(x) = \frac{10 \sin x}{x}$.



first derivative:

critical numbers:

critical points:

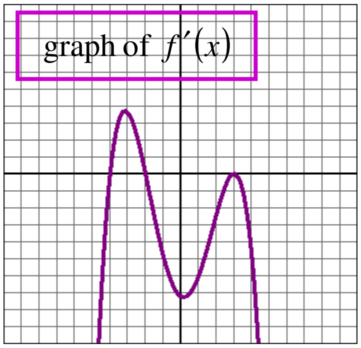
interval(s) increasing:

interval(s) decreasing:

extrema (maximum or minimum):

vertical asymptote:

horizontal asymptote:



Example J: Without knowing the function itself, describe the behavior of its graph only using information provided by its first derivative. The graph to the left is a graph of $f'(x)$.

critical numbers:

Note that, since we don't have a formula for *f*, we cannot determine y-coordinates of the critical points.

interval	$x < -4$	$x = -4$	$-4 < x < -2$
value of f'			

Since the first derivative (slope of *f*)…

interval	$-4 < x < -2$	$x = -2$	$-2 < x < 3$
value of f'			

Since the first derivative (slope of *f*) …

interval	$-2 < x < 3$	$x = 3$	$3 < x$
value of f'			

Since the first derivative (slope of *f*) …

interval(s) increasing:

interval(s) decreasing:

Putting all of the information above together, we can draw a preliminary sketch of the graph for *f*.