

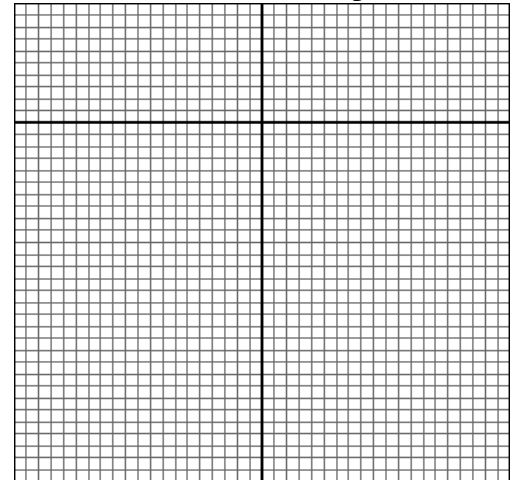
# Calculus 130, section 5.4 Sketching a Curve

notes by Tim Pilachowski

Sections 5.1-5.3 examined first and second derivative attributes of a graph, and in most cases we had the graph to consult as we went along. Now we get to begin with the equation and work toward its graph. Along with first and second derivative tests, we'll include intercepts and asymptotes (where possible) to our consideration.

Example A: Sketch the graph of  $f(x) = x^3 - 3x^2 - 9x + 1$ .

Using the factors of  $f'$  we can determine the slope of the curve on the intervals on either side of and in between these two critical values to determine whether these values represent maxima, minima, or points of inflection.



The factors of  $f''$  enable us to determine concavity.



y-intercept :

Since  $f(x) = x^3 - 3x^2 - 9x + 1$  is not easily factorable, we'll bypass locating  $x$ -intercepts for this graph. (There is a method, called Newton-Raphson, which would allow us to approximate the values, but it's not part of the syllabus for this course.)

asymptotes?

Example B: Sketch the graph of  $f(x) = (1 - 2x)^3$ .

Using the factors of  $f'$  we can determine the slope of the curve on the intervals on either side of the critical value, and thus determine whether this value represents a maximum, minimum, or point of inflection.

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The factors of  $f''$  enable us to determine concavity.

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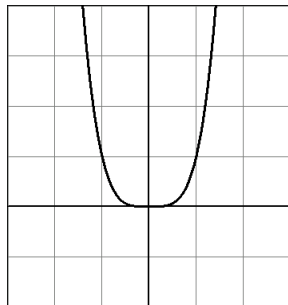
y-intercept :

x-intercept(s) :

asymptotes?

Example C. Find an example for which  $f'' = 0$  does *not* imply a point of inflection.

A table of signs indicates the nature of the curve at the origin:



interval			
sign of $y$			
sign of $\frac{dy}{dx}$			
sign of $\frac{d^2y}{dx^2}$			
behavior of graph			

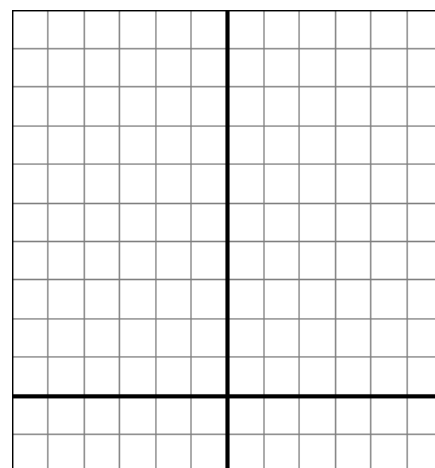
$f' > 0$  implies increasing function; increasing function does not imply  $f' > 0$  at all points, e.g.  $y = x^3$ .

$f'' > 0$  implies a graph is concave up; a graph concave up does not imply  $f'' > 0$  at all points, e.g.  $y = x^4$ .

Similar statements can be made with regard to decreasing functions and graphs that are concave down.

Example D: Sketch the graph of  $h(x) = e^{x^2-x}$ .

Find first derivative and set it equal to 0:



Use  $f''$  to determine concavity:

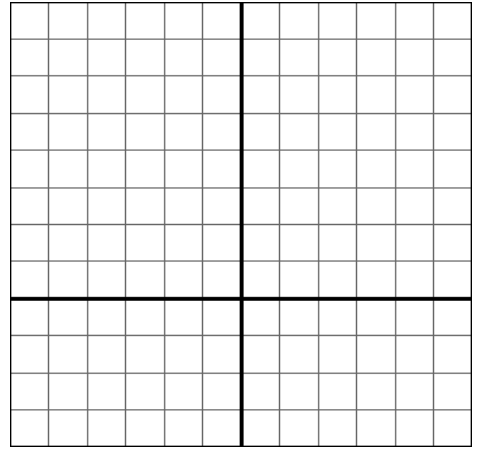
y-intercept :

x-intercept(s) :

asymptotes?

Example E: Sketch the graph of  $f(x) = 5x + e^{-2x}$ .

Find first derivative and set it equal to 0:



Use  $f''$  to determine concavity:

We conclude that the point  $\left(-\frac{1}{2} \ln \frac{5}{2}, \frac{5}{2} - \frac{5}{2} \ln \frac{5}{2}\right)$  is an absolute minimum.

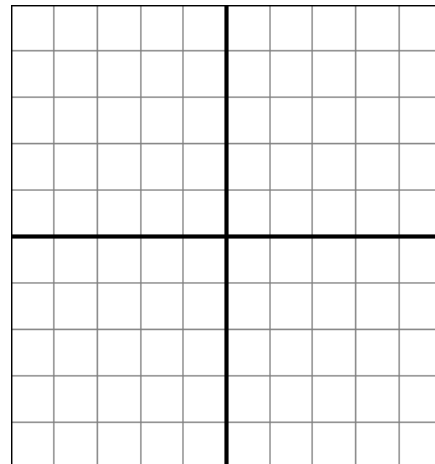
y-intercept :

x-intercept(s) :

asymptotes?

Example F: Sketch the graph of  $f(x) = \frac{3x+5}{2x^2+x-3}$ .

For this function, we'll start with asymptotes.



y-intercept :

x-intercept(s) :

extrema:

concavity:

Example G: Sketch the graph of  $f(x) = x^3 \ln|x|$ .

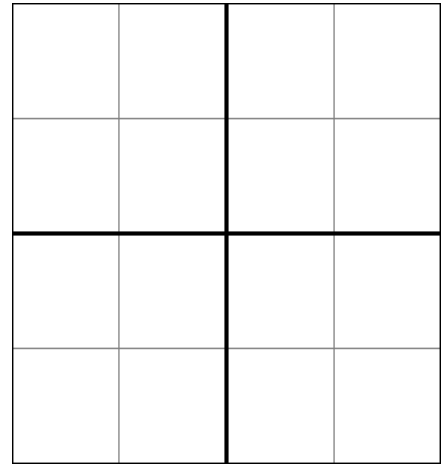
asymptotes.

y-intercept :

x-intercept(s) :

extrema:

concavity:



Example H: Sketch the graph of  $h(x) = [\ln x]^5$ .

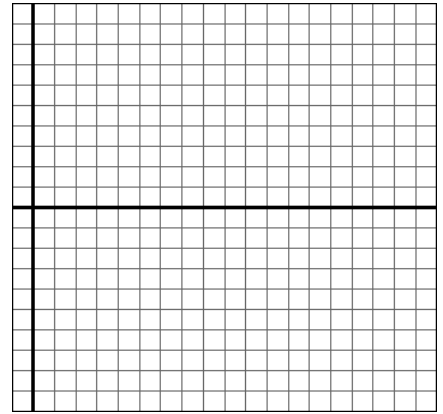
asymptotes.

y-intercept :

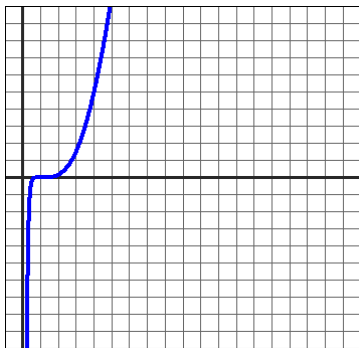
x-intercept(s) :

extrema:

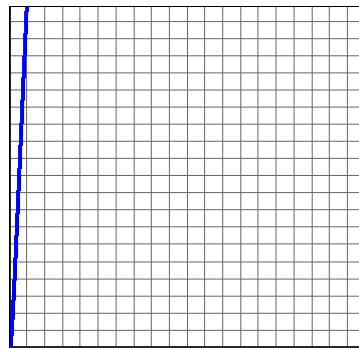
concavity:



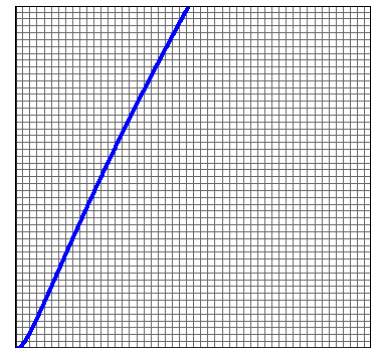
The point of inflection at  $(e^4, 1024) \approx (54.598, 1024)$  is far outside the standard window on a graphing calculator. One could zoom out a good bit, or use different scales for  $x$ - and  $y$ -axes—but then the other characteristics of the graph would be obscured. Note the three versions of the graph pictured below.



window  $[-1, 19]$  by  $[-10, 10]$



window  $[-1, 1999]$  by  $[-10, 1990]$



window  $[-1, 499]$  by  $[-10, 4990]$

Example I: Sketch the graph of  $f(x) = x + \sin x$ .

asymptotes.

y-intercept :

x-intercept(s) :

extrema:

concavity:

