

Calculus 130, section 6.2 Applications of Extrema

notes by Tim Pilachowski

Example A: Public health officials use rates of change to quantify the spread of an epidemic into an equation, which they then use to determine the most effective measures to counter it. A recent measles epidemic followed the equation $y = 45t^2 - t^3$ where y = the number of people infected and t = time in days.

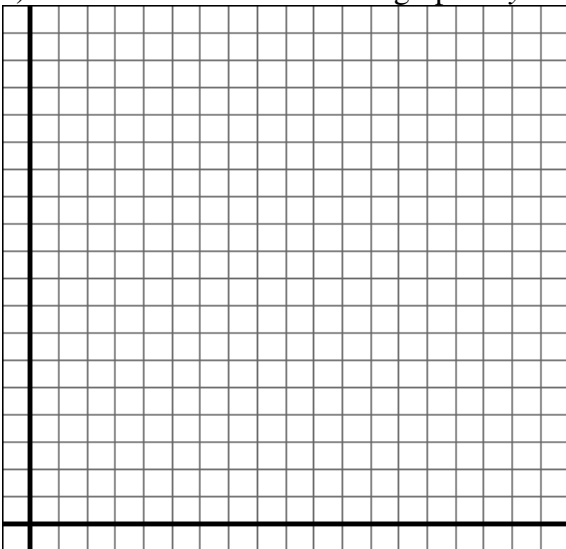
a) What is the domain of this function? *Answer:* $0 \leq t \leq 45$ days

b) How many people are infected after 5 days? *Answer:* 1000 people

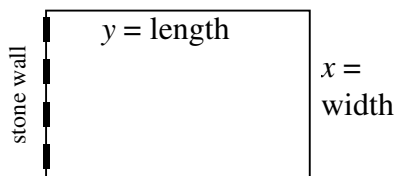
c) What is the rate of spread after 5 days? *Answer:* 375 new cases per day

d) After how many days does the number of cases reach its maximum? *Answer:* 30 days

e) Use the above to sketch the graph of y .



One prime goal in business is to keep costs to a minimum—rent or mortgage, insurance, salaries, utility bills, expense for purchasing material, advertising, etc. In the example below there will be a function which we want to minimize or maximize: the *objective function*. The limitations of reality become the *constraint function*.



Example B: Farmer Al needs to fence in 800 yd^2 , with one wall being made of stone which costs \$24 per yard, and the other three sides being wire mesh which costs \$8 per yard. What dimensions will minimize cost?
Answer: 40 yd long by 20 yd wide

Example C: Here's another optimization that involves a minimum. The fuel, maintenance and labor costs (in dollars per mile) of operating a truck on an interstate highway are described as a function of the truck's velocity (miles per hour) by the algebraic rule $C(v) = 78 + 1.2v + 5880v^{-1}$. What speed should the driver maintain on a 600 mile haul to minimize costs? *Answer:* 70 mph.

Example D: *In this example, it will be important to know which amount is being optimized.* An efficiency study of the 8 am to noon shift at a factory shows that the number of units, N , produced by an average worker t hours after 8 am is modeled by the function $N(t) = -t^3 + 9t^2 + 12t$. Note that the domain is $0 \leq t \leq 4$.

Question 1: At what time is the *number of units produced* at its peak? *Answer: noon*

Question 2: At what time is the *rate of production* at its peak? *Answer: 11 am*

Example E (logistic growth curve): Fish population as a function of time is given by $P(t) = \frac{80}{2 + 3e^{-10t}}$. Sketch the graph.

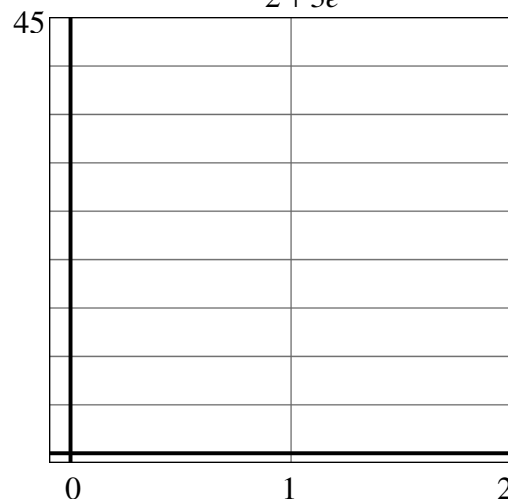
domain:

y-intercept:

x-intercept:

maxima/minima?

point of inflection?



The grid above is in a window $[-0.1, 2, 1]$ by $[-1, 45, 5]$.

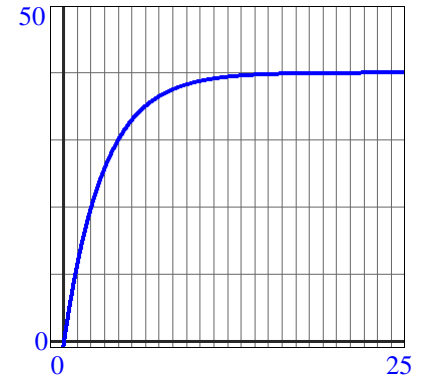
To the left of the point of inflection the graph is concave up; to the right it is concave down.

asymptotes?

What is the theoretical “upper limit” according to this model?

Note how this logistic growth curve is very steep then seems to level off quickly.

Example F: The number of units a new worker can produce on an assembly line after t days on the job is given by the formula $N(t) = 40 - 40e^{-0.35t}$. This function is called a *learning curve*. a) How many units can the worker make when she or he first begins? b) What is the maximum number he or she can be expected to make? c) Estimate how long it will take for the worker to be able to produce 30 units. d) What is the worker's rate of production? *Answers:* 0 units; 40 units; $\frac{20 \ln(4)}{7} \approx 4$ days; $14e^{-0.35t}$ units per day



Example G: In an educational psychology study a researcher determines that the number of new historical dates a person can remember after t days is given by the formula $H(t) = 125 \left(\frac{1+e}{1+e^t} \right)$, $t \geq 1$. This function is called a *forgetting curve*. a) How many dates can a person remember on day 1? b) Estimate how long it will take for a person to remember 25 dates. c) At what rate is the person forgetting? **Note:** $(1+e)$ is a number, and is treated in the same way as the constant coefficient 125.

Answers: 125 dates; $\ln(4 + 5e) \approx 3$ days; $\frac{-125(1+e)*e^t}{(1+e^t)^2}$ dates per day

