Calculus 130, section 7.5 Integrals of Trigonometric Functions

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Example A: Find $\int \sin t \, dt$. *answer*: $-\cos t + C$

Example A extended: Find the area under the curve $y = \sin t$ from $t = 0$ to $t = \frac{3\pi}{2}$. *answer*: 3

Example B: Evaluate $\int \cos\left(9t + \frac{\pi}{2}\right) dt$. *answer*: $\frac{1}{9}\sin\left(9t + \frac{\pi}{2}\right) + C$

Example C: Determine $\int \tan^2 t \, dt$. *answer*: $\tan t - t + C$

In Lecture 7.2, when we introduced integration by substitution, we had some general forms of integrals to look for,

$$\int u^n \, du, \qquad \int e^u \, du, \qquad \int \frac{1}{u} \, du, \qquad \int \ln(x) * \frac{1}{x} \, dx = \int u \, du,$$

along with some hints: When using substitution in an integral involving polynomials, it is usually most productive to let u = the function with the higher exponent. When using substitution in an integral involving ln(*a function)*, it is usually most productive to let u = ln(*a function)*. When using substitution in an integral involving $e^{(\text{exponent function})}$, it is usually most productive to let u = (exponent function).

In general, look for an "inside" function whose derivative appears as a factor elsewhere in the integral.

Example D: $\int x^2 \cos\left(x^3\right) dx$. *answer*: $\frac{1}{3}\sin\left(x^3\right)+C$

Example E: $\int (2x-1)\sec^2\left(x^2-x+1\right) dx$. *answer*: $\tan\left(x^2-x+1\right)+C$

Example F: $\int \sin^2(4x)\cos(4x)\, dx$. *answer*: $\frac{1}{12}\sin^3(4x)+C$

Example G: $\int \tan x\, dx$. *answer*: $-\ln|\cos x|+C=\ln|\sec x|+C$

Example H: $\int e^x \sec^2\left(e^x+1\right) dx$. *answer*: $\tan\left(e^x+1\right)+C$