Math 130 Exam 1 Sample 1 Solutions

1. (a) We rewrite with the same base:

$$\begin{aligned} 25^{x+1} &= 125^{2-x} \\ (5^2)^{x+1} &= (5^3)^{2-x} \\ 5^{2x+2} &= 5^{6-3x} \\ 2x+2 &= 6-3x \\ 5x &= 4 \\ x &= 4/5 \end{aligned}$$

(b) We have:

$$\begin{aligned} \ln\sqrt{\frac{ab^3}{e^2}} &= \ln\left(\frac{ab^3}{e^2}\right)^{1/2} \\ &= \frac{1}{2}\ln\frac{ab^3}{e^2} \\ &= \frac{1}{2}\left[\ln ab^3 - \ln e^2\right] \\ &= \frac{1}{2}\left[\ln a + \ln b^3 - 2\right] \\ &= \frac{1}{2}\left[\ln a + 3\ln b - 2\right] \\ &= \frac{1}{2}\left[3 + 3(2) - 2\right] \\ &= \frac{1}{2}\left[7\right] \\ &= 7/2 \end{aligned}$$

(c) We know that the population follows

$$A = A_0 e^{0.05t}$$

Quadrupling means we should have four times A_0. Thus we want to solve

$$\begin{aligned} A_0 e^{0.05t} &= 4A_0 \\ e^{0.05t} &= 4 \\ 0.05t &= \ln 4 \\ t &= 20\ln 4 \approx 27.72 \text{ years.} \end{aligned}$$

2. (a) We know we have

$$N(t) = 6e^{kt}$$

We know when $t = 24$ we have $N(24) = 3$ and so

$$\begin{aligned} 6e^{24k} &= 3 \\ e^{24k} &= 0.5 \\ 24k &= \ln 0.5 \\ k &= \frac{1}{24}\ln 0.5 \end{aligned}$$

and so

$$N(t) = 6e^{\left(\frac{1}{24}\ln 0.5\right)t}$$

(b) We use the AROC formula:

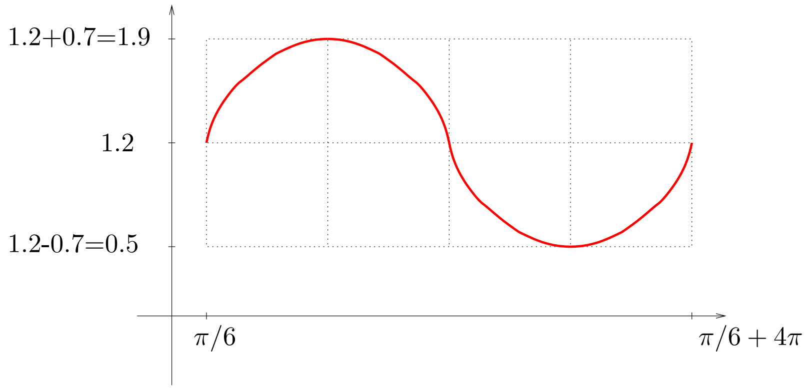
$$\begin{aligned} \text{AROC} &= \frac{N(24) - N(0)}{24 - 0} \\ &= \frac{6e^{\left(\frac{1}{24}\ln 0.5\right)24} - 6e^{\left(\frac{1}{24}\ln 0.5\right)0}}{24} \\ &= \frac{6e^{\ln 0.5} - 6e^{0}}{24} \\ &= \frac{6(0.5) - 6(1)}{24} \\ &= \frac{-3}{24} \\ &= -0.125 \approx -0.13 \end{aligned}$$

The units is thousands of bacteria per hour.

(c) We want the time so that $N(t) = 0.5$:

$$\begin{aligned} 6e^{\left(\frac{1}{24}\ln 0.5\right)t} &= 0.5 \\ e^{\left(\frac{1}{24}\ln 0.5\right)t} &= 1/12 \\ \left(\frac{1}{24}\ln 0.5\right)t &= \ln(1/12) \\ t\ln 0.5 &= 24\ln(1/12) \\ t &= \frac{24\ln(1/12)}{\ln 0.5} \approx 86.04 \text{ hours.} \end{aligned}$$

3. (a) The amplitude is 0.7. The period is $2\pi/(1/2) = 4\pi$. The phase shift (start of period) is $\pi/6$. There is also a vertical shift up by 1.2:



(b) The maximum wave height occurs one-quarter of the way through the period.

$$t = \pi/6 + (1/4)(4\pi)$$

(c) The height is 1.9.

4. (a) We have:

$$\begin{aligned}
\lim_{x\to 7} \sqrt{\frac{x-1}{x^2+5}} &= \sqrt{\frac{7-1}{7^2+5}} \\
&= \sqrt{\frac{6}{54}} \\
&= \sqrt{\frac{1}{9}} \\
&= \frac{1}{3}
\end{aligned}$$

(b) We have:

$$\begin{aligned}
\lim_{x\to 2} \frac{x^2-4}{x^2-7x+10} &= \lim_{x\to 2} \frac{(x-2)(x+2)}{(x-2)(x-5)} \\
&= \lim_{x\to 2} \frac{x+2}{x-5} \\
&= \frac{2+2}{2-5} \\
&= -\frac{4}{3}
\end{aligned}$$

(c) We have:

$$\begin{aligned}
\lim_{x\to\infty} \frac{x^3+x}{x^2-x+1} &= \lim_{x\to\infty} \frac{x^3+x}{x^2-x+1} \cdot \frac{1/x^3}{1/x^3} \\
&= \lim_{x\to\infty} \frac{1+1/x^2}{1/x-1/x^2+1/x^3}
\end{aligned}$$

Since the numerator approaches 1 and the denominator approaches 0 we know the limit does not exist.

5. (a) We have

$$\begin{aligned} f'(2) &= \lim_{h\to 0} \frac{f(2+h)-f(2)}{h} \\ &= \lim_{h\to 0} \frac{5(2+h)-5(2)}{h} \\ &= \lim_{h\to 0} \frac{5h}{h} \\ &= \lim_{h\to 0} 5 \\ &= 5 \end{aligned}$$

(b) We have

$$\begin{aligned} g'(x) &= \lim_{h\to 0} \frac{g(x+h)-g(x)}{h} \\ &= \lim_{h\to 0} \frac{\big[(x+h)^2-3(x+h)\big]-\big[x^2-3x\big]}{h} \\ &= \lim_{h\to 0} \frac{x^2+2xh+h^2-3x-3h-x^2+3x}{h} \\ &= \lim_{h\to 0} \frac{2xh+h^2-3h}{h} \\ &= \lim_{h\to 0} 2x+h-3 \\ &= 2x-3 \end{aligned}$$

(c) We have

$$\begin{aligned} \text{Derivative} &= \lim_{h\to 0} \frac{4/(x+h)-4/x}{h} \\ &= \lim_{h\to 0} \frac{4/(x+h)-4/x}{h} \cdot \frac{x(x+h)}{x(x+h)} \\ &= \lim_{h\to 0} \frac{4x-4(x+h)}{hx(x+h)} \\ &= \lim_{h\to 0} \frac{-4h}{hx(x+h)} \\ &= \lim_{h\to 0} \frac{-4}{xx+h} \\ &= \frac{-4}{x^2} \end{aligned}$$