Math 130 Exam 1 Sample 2 Solutions

1. (a) We have

$$\begin{aligned} 16^{3-x} &= 32^x \\ (2^4)^{3-x} &= (2^5)^x \\ 2^{12-4x} &= 2^{5x} \\ 12-4x &= 5x \\ 9x &= 12 \\ x &= 4/3 \end{aligned}$$

(b) We have

$$\begin{aligned} \log\sqrt{600} &= \log 600^{1/2} \\ &= \frac{1}{2}\log 600 \\ &= \frac{1}{2}(\log 2 + \log 3 + \log 100) \\ &= \frac{1}{2}(a+b+2) \end{aligned}$$

(c) The AROC is

$$\begin{aligned} \text{AROC} &= \frac{f(5)-f(3)}{5-3} \\ &= \frac{(5^2+2^5)-(3^2+2^3)}{2} \\ &= \frac{57-17}{2} \\ &= 20 \end{aligned}$$

2. (a) We know that when $t = 12$ we have $A = 2A_0$ so

$$\begin{aligned} A_0 e^{k(12)} &= 2A_0 \\ e^{12k} &= 2 \\ 12k &= \ln 2 \\ k &= \frac{1}{12}\ln 2 \end{aligned}$$

I'm going to approximate this as $k = 0.0578$ for the remaining problems.

(b) Since we know $A = 300e^{0.0578t}$ we have after five years $A = 300e^{0.0578(5)} \approx 401$ monkeys.

(c) We need

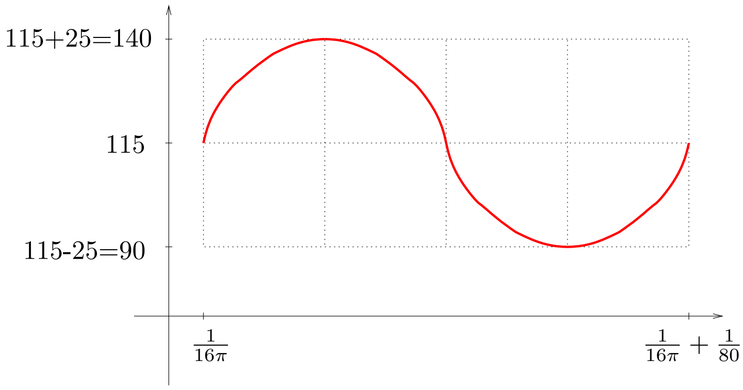
$$\begin{aligned} 300e^{0.0578t} &= 1000 \\ e^{0.0578t} &= 10/3 \\ 0.0578t &= \ln(10/3) \\ t &= \frac{\ln(10/3)}{0.0578} \approx 20.83 \text{ years} \end{aligned}$$

3. (a) This is easiest if we factor out the 160π first to get

$$p(t) = 115 + 25 \sin 160\pi \left(t - \frac{10}{160\pi} \right)$$

Then we have:

- The phase shift (start of the period) is at $\frac{10}{160\pi} = \frac{1}{16\pi}$.
- The period length is $\frac{2\pi}{160\pi} = \frac{1}{80}$.
- The amplitude is 25.
- There is a vertical shift of 115.



(b) The minimum pressure occurs three-fourths of the way through the period, so at

$$\frac{1}{16\pi} + \frac{3}{4} \cdot \frac{1}{80} \text{ minutes.}$$

(c) This pressure is 90 mmHg.

4. (a) We have the following table:

x	-0.1	-0.01	0	0.01	0.1
$(e^{2x}-1)/x$	1.8127	1.9801	$\ddot{\frown}$	2.0201	2.2140

It looks like $\lim\limits_{x\to 0} \frac{e^{2x}-1}{x} = 2.$

(b) We have:

$$\begin{aligned}
\lim_{x\to 0} \frac{x^2+x}{x^2-x} &= \lim_{x\to 0} \frac{x(x+1)}{x(x-1)} \\
&= \lim_{x\to 0} \frac{x+1}{x-1} \\
&= \frac{1}{-1} \\
&= -1
\end{aligned}$$

(c) We have

$$\begin{aligned}
\lim_{x\to\infty} \sqrt{\frac{x^2+x}{4x^2-x+1}} &= \lim_{x\to\infty} \sqrt{\frac{x^2+x}{4x^2-x+1} \cdot \frac{1/x^2}{1/x^2}} \\
&= \lim_{x\to\infty} \sqrt{\frac{1+1/x}{4-1/x+1/x^2}} \\
&= \sqrt{\frac{1+0}{4-0+0}} \\
&= \frac{1}{2}
\end{aligned}$$

5. (a) We would have $f'(3) = 1.4$ because $f'(3)$ is the slope of the tangent line and the tangent line is $y = 1.4x + 4$ which has slope 1.4.

(b) We have:

$$
\begin{aligned}
f'(2) &= \lim_{h\to 0} \frac{f(2+h) - f(2)}{h} \\
&= \lim_{h\to 0} \frac{\left[5(2+h) - (2+h)^2\right] - \left[5(2) - 2^2\right]}{h} \\
&= \lim_{h\to 0} \frac{10 + 5h - (4 + 4h + h^2) - 6}{h} \\
&= \lim_{h\to 0} \frac{h - h^2}{h} \\
&= \lim_{h\to 0} 1 - h \\
&= 1
\end{aligned}
$$

(c) We have a point, that's $(3, g(3)) = \left(3, \sqrt{7}\right)$. We just need the slope. The slope is

$$
\begin{aligned}
g'(3) &= \lim_{h\to 0} \frac{g(3+h) - g(3)}{h} \\
&= \lim_{h\to 0} \frac{\sqrt{2(3+h)+1} - \sqrt{7}}{h} \\
&= \lim_{h\to 0} \frac{\sqrt{2h+7} - \sqrt{7}}{h} \\
&= \lim_{h\to 0} \frac{\sqrt{2h+7} - \sqrt{7}}{h} \cdot \frac{\sqrt{2h+7} + \sqrt{7}}{\sqrt{2h+7} + \sqrt{7}} \\
&= \lim_{h\to 0} \frac{2h + 7 - 7}{h\left(\sqrt{2h+7} + \sqrt{7}\right)} \\
&= \lim_{h\to 0} \frac{2h}{h\left(\sqrt{2h+7} + \sqrt{7}\right)} \\
&= \lim_{h\to 0} \frac{2}{\sqrt{2h+7} + \sqrt{7}} \\
&= \frac{2}{2\sqrt{7}} \\
&= \frac{1}{\sqrt{7}}
\end{aligned}
$$

Thus the equation is

$$
y - \sqrt{7} = \frac{1}{\sqrt{7}}(x - 3)
$$