**Math 130 Exam 2 Sample 1 Solutions**

1. (a) We have

$$\begin{aligned}\frac{d}{dt}\,(t-2)^2(t+1) &= 2(t-2)(t+1)+(t-2)^2\\ &= 2(t^2-t-2)+(t^2-4t+4)\\ &= 2t^2-2t-4+t^2-4t+4\\ &= 3t^2-6t\end{aligned}$$

(b) $\frac{d}{dx}\ \log_3(x^2-3x+1) = \frac{1}{\ln 3}\frac{2x-3}{x^2-3x+1}$

(c) $\frac{d}{dx}\ e^{2-\ln(x+1)} = e^{2-\ln(x+1)}\left[-\frac{1}{x+1}\right]$

(d) $\frac{d}{dx}\ \frac{2^x(x^7+3x-1)}{x-5} = \frac{\left[(\ln 2)2^x(x^7+3x-1)+2^x(7x^6+3)\right](x-5)+2^x(x^7+3x-1)}{(x-5)^2}$

2. (a) The point is $x=9$ and $y=9\log_3 9=9(2)=18$. Since

$$f'(x)=\log_3 x + x\cdot\frac{1}{\ln 3}\cdot\frac{1}{x}=\log_3 x+\frac{1}{\ln 3}$$

we have slope $f'(9)=\log_3 9+\frac{1}{\ln 3}=2+\frac{1}{\ln 3}$ so the line has equation

$$y-18=\left(2+\frac{1}{\ln 3}\right)(x-9)$$

(b) Observe that

$$\begin{aligned}f(x) &= x\ln x\\ f'(x) &= \ln x + x\left(\frac{1}{x}\right)\\ &= \ln x+1\\ f''(x) &= \frac{1}{x}\end{aligned}$$

Inflection points can only occur when the concavity changes and the concavity can change only when $f''(x)=0$ or undefined. However $f''(x)$ is never 0 and is undefined only at $x=0$ but the function is not defined there. Therefore there are no inflection points.

3. (a) We have $P(3)=3+2\sqrt{5(3)+7}=3+2\sqrt{22}\approx 12.38$ and since $P(t)=3+2(5t+7)^{1/2}$ we have

$$P'(t)=(5t+7)^{-1/2}(5)=\frac{5}{\sqrt{5t+7}}$$

and so $P'(3)=\frac{5}{\sqrt{22}}\approx 1.07$.

Therefore after 3 weeks 12.38 people have joined and are joining at a rate of 1.07 per week.

(b) We have $V(0.01)=3+0.05\sin\left(160\pi(0.01)-\frac{\pi}{2}\right)\approx 2.98$ and then we have

$$V'(t)=0.05\cos\left(160\pi t-\frac{\pi}{2}\right)(160\pi)$$

and so $V'(0.01)=0.05\cos\left(160\pi(0.01)-\frac{\pi}{2}\right)\approx -23.90$.

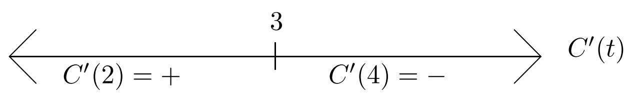
Therefore after 0.01 minutes the person's lungs contain 2.98 liters of air which is decreasing at 23.90 liters per minute.

4. (a) Since

$$C(t) = \frac{5t}{t^2+9}$$
$$C'(t) = \frac{5(t^2+9) - 5t(2t)}{(t^2+9)^2}$$
$$= \frac{45 - 5t^2}{(t^2+9)^2}$$

This is never undefined (the numerator is always positive) and equals zero when $45-5t^2 = 0$ which occurs at $t = \pm 3$.

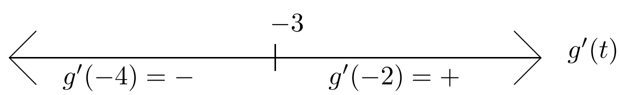
We draw a sign chart to find when this is positive. We only care about $t \geq 0$ so we get:




Thus the largest interval is $(0, 3)$.

(b) We have $g'(x) = \frac{4(x+3)^3}{(x+3)^4+32}$. This is never undefined and equals zero when $4(x+3)^3 = 0$ or $x = -3$.

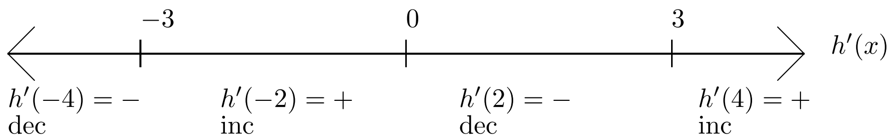
We draw a sign chart to find if this is a relative maximum or a relative minimum.




Thus there is a relative minimum at $x = -3$.

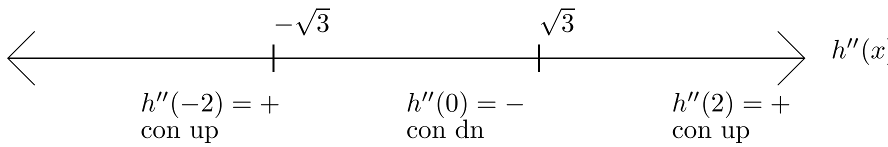
5. The $y$-intercept is at $h(0) = 5$ so at $(0, 5)$.

We have $h'(x) = 4x^3 - 36x = 4x(x^2 - 9) = 4x(x + 3)(x - 3)$ which is never undefined and equals 0 when $x = 0, -3, 3$. We draw a sign chart:



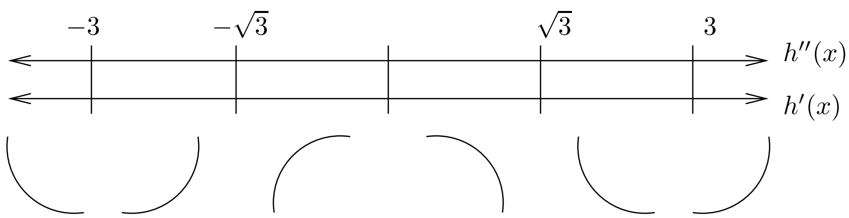

Therefore we have a relative minimum at $(-3, h(-3)) = (-3, -76)$, a relative maximum at $(0, h(0)) = (0, 5)$ and a relative minimum at $(3, h(3)) = (3, -76)$.

We have $h''(x) = 12x^2 - 36 = 12(x^2 - 3)$ which is never undefined and equals 0 when $x^2 - 3 = 0$ or $x = \pm\sqrt{3}$. We draw a sign chart:




Therefore we have inflection points at $(-\sqrt{3}, h(-\sqrt{3})) = (-\sqrt{3}, -40)$ and $(\sqrt{3}, h(\sqrt{3})) = (\sqrt{3}, -40)$.

If we overlay the sign charts we get:




The graph is therefore: