Math 130 Exam 2 Sample 2 <u>SOLUTIONS</u>

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1. (a) quotient rule: $\frac{d}{dt}\left(\frac{t+1}{2t-3}\right)=\frac{(2t-3)(1)-(t+1)(2)}{(2t-3)^2}=\frac{2t-3-2t-2}{(2t-3)^2}=\frac{-5}{(2t-3)^2}$

1. (b) chain rule: $\frac{d}{dx}\left[\left(x^2+4x-1\right)^{1/2}\right]=\frac{1}{2}\left(x^2+4x-1\right)^{-1/2}(2x+4)=(x+2)\left(x^2+4x-1\right)^{-1/2}$

1. (c) log properties first, then chain rule: $\frac{d}{dq}\left[\ln\left(\frac{q-1}{q^2-e^q}\right)\right]=\frac{d}{dq}\left[\ln(q-1)-\ln\left(q^2-e^q\right)\right]=\frac{1}{q-1}-\frac{2q-e^q}{q^2-e^q}$

1. (d) product rule and chain rule: $\frac{d}{ds}\left[\left(s^2+1\right)\cos\left(3s^2+(s-1)^{1/2}\right)\right]$

$$=\left(s^2+1\right)\left[-\sin\left(3s^2+(s-1)^{1/2}\right)*\left(6s+\frac{1}{2}(s-1)^{-1/2}\right)\right]+(2s)\cos\left(3s^2+(s-1)^{1/2}\right)$$

1. (e) exponent property then chain rule:

$$\frac{d}{dt}\left[\left(e^{\ln 2}\right)^{4x^2+x-1}\right]=\frac{d}{dt}\left(e^{(\ln 2)*\left(4x^2+x-1\right)}\right)=e^{(\ln 2)*\left(4x^2+x-1\right)}*(\ln 2)(8x+1)=2^{4x^2+x-1}*(\ln 2)(8x+1)$$

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2. (a) $f(4)=\log_2\left(4^2-8\right)=\log_2(8)=3 \;\Rightarrow\;$ point $(x, y) = (4, 3)$

$$f(x)=\left(\frac{1}{\ln 2}\right)\left[\ln\left(x^2-8\right)\right] \;\Rightarrow\; f'(x)=\left(\frac{1}{\ln 2}\right)\left[\frac{2x}{x^2-8}\right] \;\Rightarrow\; f'(4)=\left(\frac{1}{\ln 2}\right)\left[\frac{2(4)}{4^2-8}\right]=\frac{1}{\ln 2}=m \text{ (slope)}$$

$$y=mx+b \;\Rightarrow\; 3=\frac{1}{\ln 2}(4)+b \;\Rightarrow\; 3-\frac{4}{\ln 2}=b \;\Rightarrow\; y=\frac{x}{\ln 2}+\left(3-\frac{4}{\ln 2}\right)$$

2. (b) Rewrite: $P(t)=3000\left(1+20e^{-0.02t}\right)^{-1}$

$$P'(t)=3000(-1)\left(1+20e^{-0.02t}\right)^{-2}\left[20\left(-0.02e^{-0.02t}\right)\right]=\frac{1200e^{-0.02t}}{\left(1+20e^{-0.02t}\right)^2}$$

$$P''(t)=\frac{\left(1+20e^{-0.02t}\right)^2\left(-24e^{-0.02t}\right)-\left(1200e^{-0.02t}\right)(2)\left(1+20e^{-0.02t}\right)\left(-0.4e^{-0.02t}\right)}{\left(1+20e^{-0.02t}\right)^4}$$

$$=\frac{\left(1+20e^{-0.02t}\right)^2\left(-24e^{-0.02t}\right)+960e^{-0.02t}\left(e^{-0.02t}\right)\left(1+20e^{-0.02t}\right)}{\left(1+20e^{-0.02t}\right)^4}$$

$$=\frac{\left(-24e^{-0.02t}\right)\left(1+20e^{-0.02t}\right)\left[\left(1+20e^{-0.02t}\right)-40e^{-0.02t}\right]}{\left(1+20e^{-0.02t}\right)^4}$$

$$=\frac{\left(-24e^{-0.02t}\right)\left(1+20e^{-0.02t}\right)\left[1-20e^{-0.02t}\right]}{\left(1+20e^{-0.02t}\right)^3}$$

Solve $1-20e^{-0.02t}=0 \;\Rightarrow\; 1=20e^{-0.02t} \;\Rightarrow\; \frac{1}{20}=e^{-0.02t} \;\Rightarrow\; \ln\left(\frac{1}{20}\right)=-0.02t$

$$\Rightarrow\; -\ln(20)=-0.02t \;\Rightarrow\; 50\ln(20)=t\approx 149.79 \text{ days}$$

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3. (a) $P(7)=\frac{300}{1+2e^{-0.02(7)}}=\frac{300}{1+2e^{-0.14}}\approx 109.54$ bacteria after 7 days

$P'(t)=300(-1)\left(1+2e^{-0.02t}\right)^{-2}\left(-0.04e^{-0.02t}\right)=\frac{12e^{-0.02t}}{\left(1+2e^{-0.02t}\right)^2} \Rightarrow P'(7)=\frac{12e^{-0.14}}{\left(1+2e^{-0.14}\right)^2}\approx 1.39$ bacteria per day is the rate of growth after 7 days

3. (b) i. After 3 hours, there are 5 mg of medicine in the patient, the amount of medicine is increasing at a rate of 0.4 mg per hour, and this rate is increasing (slope is getting steeper).

ii. After 3 hours, there are 2 mg of medicine in the patient, the amount of medicine is decreasing at a rate of 0.4 mg per hour, and this rate is increasing (slope is getting less steep).

iii. After 3 hours, there are 10 mg of medicine in the patient, the amount of medicine is increasing at a rate of 1.1 mg per hour, and this rate is decreasing (slope is getting less steep).

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4. (a) Note that the domain of f is $(-\infty, \infty)$.

$$\frac{d}{dx}\left[\left((x-4)^2+5\right)^{1/2}\right]=\frac{1}{2}\left((x-4)^2+5\right)^{-1/2}*2(x-4)=\frac{x-4}{\sqrt{(x-4)^2+5}}$$

Solve $x-4>0$: The function is increasing on the open interval $(4, \infty)$.
Solve $x-4<0$: The function is increasing on the open interval $(-\infty, 4)$.

4. (b) Rewrite $g(x)=x^2+16x^{-1}$. Then $g'(x)=2x-16x^{-2}$.

Solve $2x-\frac{16}{x^2}=0 \Rightarrow 2x^3-16=0 \Rightarrow x^3=8 \Rightarrow x^3=2$

Critical numbers: $x = 0$ (derivative is undefined) and $x = 2$ (derivative = 0)

interval	$-\infty<x<0$	$x=0$	$0<x<2$	$x=2$	$2<x<\infty$
$g'(x)$ is…	(–)	undefined	(–)	0	(+)

The function has a relative minimum at $x = 2$.

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5. y-intercept: $f(0)=\left(\frac{0+2}{0-1}\right)^2=4 \Rightarrow (0, 4)$

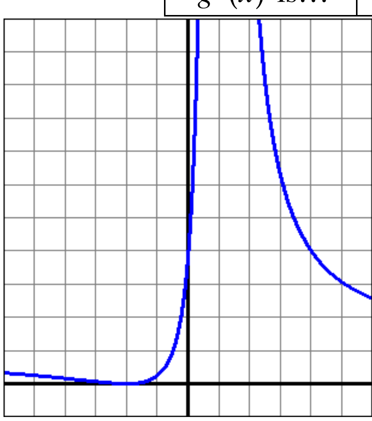
increasing/decreasing/extrema: $f'(x)=0=-6x-12 \Rightarrow x=-2$

interval	$-\infty<x<0$	$x=-2$	$-2<x<1$	$x=1$	$1<x<\infty$
$g'(x)$ is…	(–)	0	(+)	undefined	(+)

The function is increasing on the intervals $(-2, 1)$ and $(2, \infty)$. The function is increasing on the interval $(-\infty, 0)$. The function has relative minimum at the point $(-2, 0)$.

concavity: $f''(x)=0=12x+42 \Rightarrow x=-\frac{42}{12}=-\frac{7}{2}=-3.5$

interval	$-\infty<x<-3.5$	$x=-3.5$	$-3.5<x<1$	$x=1$	$1<x<\infty$
$g'(x)$ is…	(–)	0	(+)	undefined	(+)



The function is concave up on the intervals $(-3.5, 1)$ and $(1, \infty)$. The function is concave down on the interval $(-\infty, -3.5)$. The function has point of inflection at $x = -3.5$.

The End