

Math 130 Final Exam Sample 1

Directions: Do not simplify unless indicated. Non-graphing calculators are permitted. Show all work as appropriate for the methods taught in this course. Partial credit will be given for any work, words or ideas which are relevant to the problem.

Please put problem 1 on answer sheet 1

1. (a) Solve the following equations:

$$(i) 64^{x-1} = 16^{2-x} \quad (ii) \log(x+5) + \log(x+2) = 1 \quad (iii) 5 - 6^{3x-1} = 1$$

- (b) Sketch one period of the graph $f(x) = -2 \sin \frac{1}{2} (x - \frac{\pi}{6})$. Mark units on the x and y -axis and mark the intercepts and the maximum and minimum with their coordinates.
- (c) A certain drug is injected into the bloodstream and decays exponentially. Suppose 0.2mL are injected and seven hours later only 0.15mL remains. Find the function which gives the amount after any time t .

Please put problem 2 on answer sheet 2

2. (a) Evaluate the following limits. Do (iii) by plugging in nearby values.

$$(i) \lim_{x \rightarrow \infty} \sqrt{\frac{x^2+2}{4x+1}} \quad (ii) \lim_{x \rightarrow 2} \frac{x^2-4}{x^2+x-6} \quad (iii) \lim_{x \rightarrow 3} \frac{|x-3|}{x-3}$$

- (b) Use the limit definition of the derivative to find $f'(3)$ for $f(x) = x^2 - 3x$.
- (c) If $L(t) = 0.2t^2 + 0.1t$ gives the average horn length for a certain species after t months. Find the average growth rate over the first year.

Please put problem 3 on answer sheet 3

3. (a) Find the following derivatives.

$$(i) \frac{d}{dx} [3x^2 + 5x] \quad (ii) f'(2) \text{ for } f(x) = (4x-2)^3(5x+1)^2 \quad (iii) \frac{d^2}{dt^2} \left[\frac{t+1}{t-3} \right]$$

- (b) Find the following derivatives.

$$(i) g'(x) \text{ for } g(x) = 7x \tan(5x+3) \quad (ii) \frac{d}{dx} [2xe^{(10x+12)}] \quad (iii) h'(1) \text{ for } h(x) = \frac{x}{\ln(2x+3)}$$

- (c) If $P(t) = 0.02t^2 + 3e^{0.01t}$ gives the population of a certain species after t days, find and interpret $P(10)$ and $P'(10)$. Give exact answers and approximations to two decimal digits.

Please put problem 4 on answer sheet 4

4. (a) Let $f(x) = \frac{1}{5}x^5 - x$.

- Find the x and y -intercepts as points.
 - Find the intervals on which $f(x)$ is increasing and on which it is decreasing. Find all relative extrema as points.
 - Find the intervals on which $f(x)$ is concave up and on which it is concave down. Find all inflection points.
 - Sketch the graph of the function. Mark the coordinates of all points found in i, ii and iii.
- (b) Suppose the amount of medication in a person's body is given by $Q(t) = te^{-At}$ where A is a positive constant and $t \geq 0$. Determine the intervals of increasing/decreasing for $Q(t)$ and describe what this means in real-world terms with a sentence or two.

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Please put problem 5 on answer sheet 5

5. (a) Find the maximum and minimum values of $f(x) = \frac{1}{16}x^2 + \frac{1}{x^2}$ on the interval $[1, 4]$.
(b) Using implicit differentiation find $\frac{dy}{dx}$ where y satisfies $xy^2 - \frac{1}{y} = y$.
(c) A bacterial culture in the shape of a semicircle grows at 3 square inches per day. How fast is the radius growing at the instant when the area is 20 square inches?

Please put problem 6 on answer sheet 6

6. (a) Determine each of the following indefinite integrals. If you use a substitution make it clear.
(i) $\int \frac{x^2 - x + 3}{x^2} dx$ (ii) $\int 4 \cot(5x + 7) dx$ (iii) $\int x\sqrt{1-x} dx$
(b) Suppose a population satisfies $P'(t) = 0.02t^2 + t + e^{0.1t}$ and $P(0) = 5$. Find $P(t)$ for any t .

Please put problem 7 on answer sheet 7

7. (a) Using a left endpoint sum approximate $\int_0^2 \sqrt{x} dx$ with $n = 4$ subintervals. Draw a picture of your graph and your rectangles.
(b) Determine each of the following definite integrals. If you use a substitution make it clear.
(i) $\int_1^2 (x-4)(x^2+1) dx$ (ii) $\int_0^1 6xe^{(x^2+1)} dx$ (iii) $\int_0^2 \frac{x}{\sqrt{x+7}} dx$
(c) If the rate of change of a certain quantity is given by $Q(t) = 50 - 0.2t^2$, how much does the quantity change from $t = 0$ to $t = 10$?

The End