Math 130 Final Exam Sample 2

Directions: Do not simplify unless indicated. Non-graphing calculators are permitted. Show all work as appropriate for the methods taught in this course. Partial credit will be given for any work, words or ideas which are relevant to the problem.

Please put problem 1 on answer sheet 1

1. (a) Solve the following equations:

$$\text{(i) } 25^{3x-1} = 5 \qquad \text{(ii) } \log_2(x-3) - 3 = 0 \qquad \text{(iii) } 5 - 6^{3x-1} = 1$$

(b) Sketch one period of the graph $f(x) = \frac{1}{2}\sin\left[2\left(x + \frac{\pi}{4}\right)\right]$. Mark units on the x and y-axis and mark the x-intercepts and the maximum and minimum with their coordinates.

(c) A certain population grows exponentially. After 3 years it reaches 1000 and after 5 years it reaches 2000. Find the growth constant k.

Please put problem 2 on answer sheet 2

2. (a) Evaluate the following limits. Do (iii) by plugging in nearby values.

$$\text{(i) } \lim_{x \to \infty} \frac{-2x^2 + 5}{x^3 + 2x + 1} \qquad \text{(ii) } \lim_{x \to 2} \frac{x^2 - 2x}{x^2 + x - 6} \qquad \text{(iii) } \lim_{x \to 0} \frac{x^2}{x^3 - 5x}$$

(b) Use the limit definition of the derivative to find $f'(7)$ for $f(x) = \sqrt{x - 3}$.

(c) Find the average rate of change of $f(x) = 2x + \frac{1}{x}$ from $x = 1$ to $x = 3$. Simplify.

Please put problem 3 on answer sheet 3

3. (a) Find the following derivatives.

$$\text{(i) } \frac{d}{dt}\left[\frac{3}{t^2} - \frac{2}{t}\right] \qquad \text{(ii) } f'(0) \text{ for } f(x) = (7 - 3x)^{-3}(x + 7)^4 \qquad \text{(iii) } \frac{d^2}{dt^2}\left[\frac{t}{2t+1}\right]$$

(b) Find the following derivatives.

$$\text{(i) } g'(x) \text{ for } g(x) = e^{-2x}\sec(7x) \qquad \text{(ii) } \frac{d}{dx}\cos^2(-3x + 1) \qquad \text{(iii) } h'(10) \text{ for } h(x) = \ln\left(\frac{x+1}{x-3}\right)$$

(c) Suppose $h(t)$ gives the population of a bacterial culture after t days. For each of the following what can you say about h, h' and h''?

 i. After 7 days there are 100 bacteria. The number of bacteria is increasing at a decreasing rate.

 ii. After 3 days there are 200 bactera. The number of bacteria is decreasing at an increasing rate.

Please put problem 4 on answer sheet 4

4. (a) Let $f(x) = x(x - 2)^3$.
 This function has $f'(x) = 2(2x - 1)(x - 2)^2$ and $f''(x) = 12(x - 2)(x - 1)$.

 i. Find the x and y-intercepts as points.

 ii. Find the intervals on which $f(x)$ is increasing and on which it is decreasing. Find all relative extrema as points.

 iii. Find the intervals on which $f(x)$ is concave up and on which is it concave down. Find all inflection points.

 iv. Sketch the graph of the function. Mark the coordinates of all points found in i, ii and iii.

(b) Sketch the graph of a function $g(x)$ which satisfies all of the following: $f(x) \geq 0$, $f'(x) > 0$ for $x < 3$, $f'(x) < 0$ for $x > 3$, $f''(x) < 0$ for $x < 5$ and $f''(x) > 0$ for $x > 5$.

Please put problem 5 on answer sheet 5

5. (a) Find the maximum and minimum values of $f(x) = 2x^3 - 3x^2 - 36x + 1$ on the interval $[0, 5]$.

 (b) Using implicit differentiation find $\frac{dy}{dx}$ where y satisfies $xy - xy^3 = \sin(xy)$.

 (c) A certain bird lays an egg in the shape of a perfect sphere. If the egg grows at 2 cubic centimeters per month how fast is the radius growing when the volume is 10 cubic centimeters?

Please put problem 6 on answer sheet 6

6. (a) Determine each of the following indefinite integrals. If you use a substitution make it clear.

 (i) $\int \frac{x-3}{x^3} \, dx$ (ii) $\int \frac{3x}{e^{-5x^2}} \, dx$ (iii) $\int x^2(1 - 2x)^5 \, dx$

 (b) A certain population starts at 1000 and has rate of change $t + e^{0.1t}$ members per year. What is the population after 5 years?

Please put problem 7 on answer sheet 7

7. (a) Using a left endpoint sum approximate $\int_1^7 \frac{1}{x} \, dx$ with $n = 6$ subintervals. Draw a picture of your graph and your rectangles.

 (b) Determine each of the following definite integrals. If you use a substitution make it clear.

 (i) $\int_1^2 x(x^3 + x + 1) \, dx$ (ii) $\int_0^{\sqrt{\pi}} x \cos(x^2) \, dx$ (iii) $\int_0^2 \frac{x}{x+1} \, dx$

 (c) If $f'(x) = x^2 - 3x$ find $f(5) - f(3)$.

The End