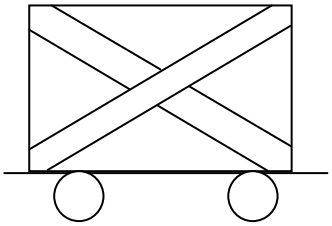
Calculus 141, section 6.4 Work

notes by Tim Pilachowski, Spring 2007

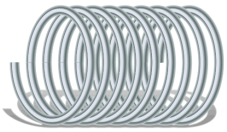
Work is essentially defined as (force exerted) times (distance traveled). When the amount of force is not constant, we can use the same sort of mathematical thinking applied to volumes in section 6.1 and to lengths of a curve in 6.2. The work resulting from a force, $F(x)$, exerted over an interval $[a, b]$ can be approximated via a series of partitions created over increasingly smaller intervals. As the number of intervals, n, approaches infinity (i.e. $\|P\|$ approaches 0), the resulting Riemann sum yields a formula:

$$W = \lim_{\|P\| \to 0} \sum_{k=1}^{n} F(t_k) * \Delta k = \int_a^b F(x)\, dx$$

Example A: A 200-pound crate is on a cart weighing 30 pounds, and is pushed for a distance of 10 feet. Assuming the cart's wheels are well-lubricated, the friction is negligible. How much work is involved? *Answer*: 2300 foot-pounds



Example B: The farther a spring is stretched from its length "at rest", the greater the force with which it pulls back. According to Hooke's law, the force exerted is proportional to the distance it is stretched, i.e. $F(x) = kx$. If a certain spring requires 10 lbs of force to stretch from rest to 1 foot, how much work is done? *Answer*: 5 ft-lbs

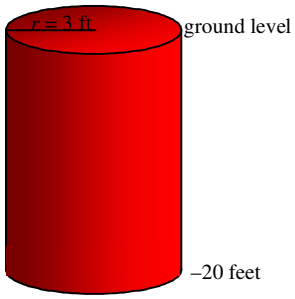


Example B extended: The same spring in example B is stretched from 1 to 2 feet. How much work is involved? *Answer*: 15 ft-lbs

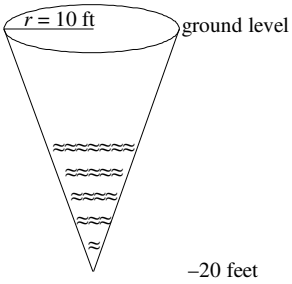
Example B extended again: How much work is involved in stretching the same spring from rest to 2 feet? *Answer*: 20 ft-lbs

Example C: What if we had another spring, and knew that it took 15 foot-pounds to stretch it to 1 foot? What is the force function $F(x)$? *Answer*: $30x$

Example D: A cylindrical well 20 feet deep and 3 feet in radius is dug. Assuming soil weighs 150 lbs per cubic foot, calculate the work W required to raise the soil to ground level. *Answer*: 270,000π ft-lbs



Example E: Similar to Example D, except that the tank is a circular right cone of height 20 feet and radius at the top of 10 feet containing 144π cubic feet of water. How much work is required to pump all the water to a point 16 feet above the ground? *Answer*: 243,000π ft-lbs



NOTE: For text question 6.4 #21, you'll need to go back to section 5.1 to review left-hand, midpoint, and right-hand sums.