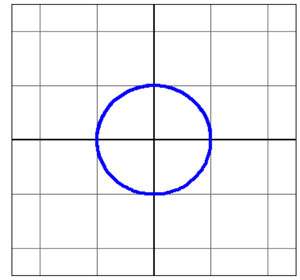
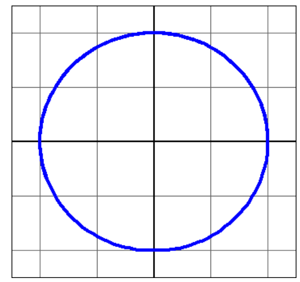
Calculus 141, section 6.7 Parametrized Curves

notes by Tim Pilachowski

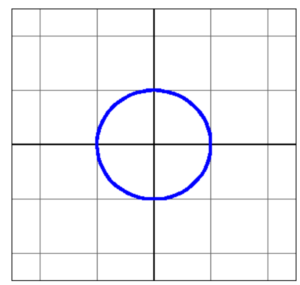


Most of the curves you have encountered in Calculus so far have been functions, but not all curves are functions. A good example is the unit circle, with equation $x^2 + y^2 = 1$. In Trigonometry, you considered the unit circle traversed counterclockwise from (1, 0) once around to where you started. The result was defining the *x*- and *y*-coordinates as $x = \cos t$ and $y = \sin t$ with t moving from 0 to 2π. In other words, while the unit circle itself cannot be expressed as y = (function of x), the x-coordinates on the circle can be expressed as a function of a parameter t, as can the y-coordinates. A more formulaic way of expressing this is to say that the unit circle can be parametrized by the function

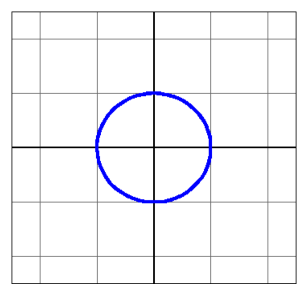
$$P(t) = (\cos t, \sin t) \qquad \text{for } 0 \le t \le 2\pi$$



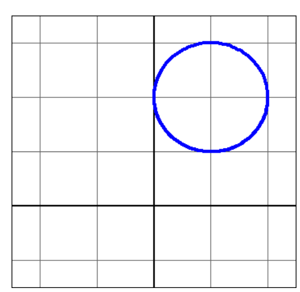
Example A: Consider the curve parametrized by $x = 2\cos t$ and $y = 2\sin t$ for t moving from 0 to 2π. *Answers*: center (0, 0), radius = 2, counterclockwise from (2, 0), once



Example B: Consider $x = \cos(2t)$ and $y = \sin(2t)$ for t moving from 0 to 2π.
Answers: center (0, 0), radius = 1, counterclockwise from (1, 0), twice

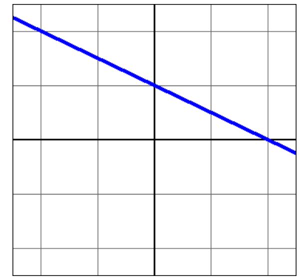


Example C: Consider the curve parametrized by $x = \sin t$ and $y = \cos t$ for t moving from 0 to 2π. **This is not the same as the unit circle considered in the introduction.**
Answers: center (0, 0), radius = 1, clockwise from (0, 1), once



Example D: Consider $x = 1 + \cos(t)$ and $y = 2 - \sin(t)$ for t moving from 0 to 2π.
Answers: center (1, 2), radius = 1, clockwise from (2, 2), once

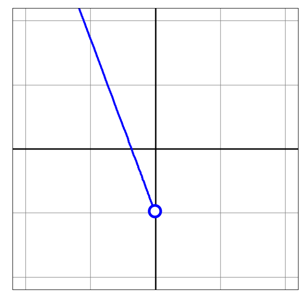
Example E: Consider $x = 2t$, $y = 1 - t$, for all t. *Answer*: $y = 1 - 0.5x$



In WebAssign, you'll take a look at a case of horizontal/vertical lines.

What do you think would happen:
If the parametric equation for x were linear and the parametric equation for y were quadratic?
If the parametric equation for x were quadratic and the parametric equation for y were linear?
If both were quadratic?

Example F: Consider $x = -e^{t-1}$, $y = e^{t} - 1$. *Answer*: $y = -ex - 1$, $x < 0$



Example G: Consider $x = t^3 - 3t^2$, $y = t^3 - 3t$. *Answer*: see graph