Calculus 141, section 7.2 The Number "*e*" and ln(*x*)

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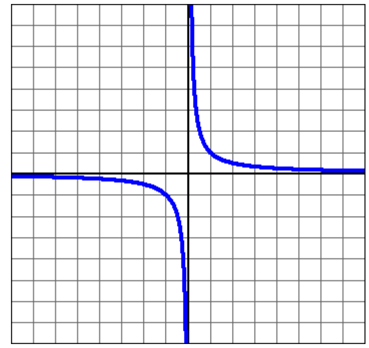


Question from 7.1: What common functions are their own inverse?

1) $f(x) = \frac{1}{x}$, since $f\left(\frac{1}{x}\right) = \frac{1}{\frac{1}{x}} = x$ for all values of $x \neq 0$. This satisfies both $f(a) = b \Rightarrow f^{-1}(b) = a$ and $f \circ f^{-1} = f^{-1} \circ f = x$ requirements to be its own inverse function. Notice, too, that the graph of $f(x) = \frac{1}{x}$ is symmetric with respect to itself across the line $y = x$.

2) $f(x) = x$ is also its own inverse; it meets the same necessary requirements.

And now for something completely different (section 7.2): Who was Euler? And why does he have a number named after him? More on this later. First, some information you'll need for homework.

By definition, the natural logarithm $\ln x = \int_1^x \frac{1}{t}\,dt$ and $\frac{d}{dx}(\ln x) = \frac{1}{x}$, both with domains of $x > 0$.

The inverse of the natural logarithm is the natural exponential function $y = e^x$, with $\frac{d}{dx}\left(e^x\right) = e^x$.

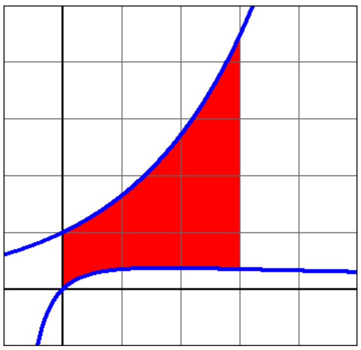
Because they are inverse functions, it will always be true that $e^{\ln x} = x$ and $\ln\left(e^x\right) = x$ for all x in the domain. All the usual differentiation rules apply: product rule, quotient rule, and chain rule.

Example A: Given $h(x) = e^{x^2 - x}$, determine the location of any relative extrema. *Answer*: $\left(\frac{1}{2}, \frac{1}{\sqrt[4]{e}}\right)$

For some homework, you'll also need implicit differentiation (section 3.6).

Example B: Given $e^{xy} = 2x + y$, find $\frac{dy}{dx}$. *Answer*: $\frac{2 - ye^{xy}}{xe^{xy} - 1}$

Example C: Find the area A of the region bounded by the graphs of $f(x) = e^{x/2}$ and $g(x) = \frac{\ln(x+1)}{x+1}$ on [0, 3].



Answer: $2e^{3/2} - \frac{(\ln 4)^2}{2} - 2$

Example D: Let $p(x) = e^x + \frac{1}{4e^x}$. Find the length of the curve L on the interval [0, 1]. *Answer*: $e - \frac{1}{4e} - \frac{3}{4}$