Calculus 141, section 8.4 Partial Fractions

notes by Tim Pilachowski

Task: Integrate a rational function. Method: Rewrite the complicated rational function as the sum of simpler rational functions, i.e. ones which are easily integrated. To do this, we have three steps:
i) Use synthetic (long) division to get the degree of the numerator less than the degree of the denominator.
ii) Factor numerator and denominator into linear and irreducible quadratic factors. Reduce, if possible.
iii) Rewrite the original function as the sum of simpler fractions. (This is the hard part.)

Example A: Find $\int \frac{2}{1-x^2}\,dx$. *Answer*: $\ln|1+x| - \ln|1-x| + C$

Example B: Find $\int \frac{6x+7}{(x+2)^2}\,dx$. *Answer*: $6\ln|x+2| + \frac{5}{x+2} + C$

Example C: Evaluate $\int \frac{2x^3-4x^2-x-3}{x^2-2x-3}\,dx$. *Answer*: $x^2 + 2\ln|x+1| + 3\ln|x-3| + C$

Example D: Evaluate $\int \frac{-2x+4}{(x^2+1)(x-1)^2}\,dx$. *Answer*: $\ln(x^2+1)+\tan^{-1}x-2\ln|x-1|-\frac{1}{x-1}+C$

Example E: Find $\int \frac{x+2}{x^2+2x+5}\,dx$. *Answer*: $\frac{1}{2}\ln(x^2+2x+5)+\frac{1}{2}\tan^{-1}\left(\frac{x+1}{2}\right)+C$

Example F: Evaluate $\int \frac{1}{y(y^2+1)}\,dy$. *Answer*: $\ln|y|-\frac{1}{2}\ln(y^2+1)+D$