Calculus 141, section 8.5 Symbolic Integration

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Back in my day (mid-to-late 1970s for high school and college) we had no handheld calculators, and the only electronic computers were mainframes at universities and government facilities. After learning the integration techniques that you are now learning, we were sent to Tables of Integrals, which listed results for (often) hundreds of simple to complicated integration results. Some could be evaluated using the methods of Calculus 1 and 2, others needed more esoteric methods. It was necessary to scan the Tables to find the form that was needed. If it was there, great! If not…

The UMCP Physics Department posts one from the textbook they use (current as of 2017) at www.physics.umd.edu/hep/drew/IntegralTable.pdf.

A similar Table of Integrals from
http://www.had2know.com/academics/table-of-integrals-antiderivative-formulas.html
is appended at the end of this Lecture outline.

Example F from 8.1: Evaluate $\int e^x \sin x \, dx$ using a Table of Integrals. *Answer*: $\frac{1}{2}e^x(\sin x - \cos x) + C$

If you remember, we had to define *I*, do a series of two integrations by parts, then solve for "*I* =".

From the Had2Know Table of Integrals below:

$$98.\ \int e^{ax}\sin(bx)\,dx =$$

$$\frac{e^{ax}}{a^2+b^2}[a\sin(bx) - b\cos(bx)]$$

Identify *a* = and *b* = , then plug those values in, and voila!

Now, with the development of handheld calculators and personal computers, much more is available to us, including software that will do all the work. The text mentions Derive, Maple, Mathematica, and MATLAB, and gives examples of Mathematica commands. You'll be using MATLAB in Math 241 and several other courses. [Be careful – software sometimes works in the complex plane, not just the real number plane!]

Example F from 8.1: Evaluate $\int e^x \sin x \, dx$ using MATLAB. *Answer*: $\frac{1}{2}e^x(\sin x - \cos x) + C$

```
>> syms x
>> int(exp(x)*sin(x))

ans =

-(exp(x)*(cos(x) - sin(x)))/2
```

Notice that 1) MATLAB assumes integration "dx" since x is the only defined variable, and 2) MATLAB does not put a "+ C" in its indefinite integrals.

Example F from 8.1 revised: Evaluate $\int_0^{\pi/3} e^x \sin x \, dx$ using MATLAB. *Answer*: $\frac{1}{2}\left(e^{\pi/3}\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)\right) + \frac{1}{2}$

```
>> int(exp(x)*sin(x),0,pi/3)

ans =

(exp(pi/3)*(3^(1/2)/2 - 1/2))/2 + 1/2
```

Notice that 1) "syms x" was not needed, since we had previously defined our variable, and 2) parentheses in the answer are very carefully placed by MATLAB.

So, why do we have to take courses like Calculus I and Calculus II to learn integration techniques if we have software that will do it for us?

Two reasons:

1)

2)

Example A: Evaluate $\int e^{-x^2}\,dx$ using MATLAB.

```
>> int(exp(-x^2))

ans =

(pi^(1/2)*erf(x))/2
```

The "`erf(x)`" is MATLAB's error function.

There are some functions whose integrals cannot be evaluated. This is why we need the numeric approximation methods of section 8.6.

Basic Formulas

1. $\int ax^n\,dx = \frac{a}{n+1}x^{n+1}$
2. $\int \frac{dx}{ax+b} = \frac{1}{a}\ln|ax+b|$
3. $\int ab^x\,dx = \frac{1}{\ln b}ab^x$
4. $\int \sin ax\,dx = \frac{-\cos ax}{a}$
5. $\int \cos ax\,dx = \frac{\sin ax}{a}$
6. $\int \tan ax\,dx = \frac{\ln|\cos ax|}{a}$
7. $\int u\,dv = uv - \int v\,du$
8. $\sin(x) = \cos(x - \frac{\pi}{2})$
9. $\sin^2(x) = 1 - \cos^2(x)$
10. $\sin(2x) = 2\sin(x)\cos(x)$
11. $\cos(2x) = 2\cos^2(x) - 1$
12. $\tan(x) = \frac{1-\cos(2x)}{\sin(2x)}$
13. $a\sin(x) + b\cos(x) =$

    $\sqrt{a^2+b^2}\sin(x + \tan^{-1}\frac{b}{a})$

Rational Functions

14. $\int \frac{x}{ax+b}\,dx = \frac{x}{a} - \frac{b}{a^2}\ln|ax+b|$
15. $\int \frac{x^2}{ax+b}\,dx = \frac{x^2}{2a} - \frac{bx}{a^2} - \frac{3b^2}{a^3} +$

    $\frac{b^2}{a^3}\ln|ax+b|$
16. $\int \frac{dx}{(ax+b)^2} = \frac{-1}{a(ax+b)}$
17. $\int \frac{x}{(ax+b)^2}\,dx = \frac{b}{a^2(ax+b)} + \frac{1}{a^2}\ln|ax+b|$
18. $\int \frac{x^2}{(ax+b)^2}\,dx = \frac{ax+b}{a^3} - \frac{b^2}{a^3(ax+b)} -$

    $\frac{2b}{a^3}\ln|ax+b|$
19. $\int \frac{dx}{x(ax+b)} = \frac{1}{b}\ln\left|\frac{x}{ax+b}\right|$
20. $\int \frac{dx}{x^2(ax+b)} = \frac{a}{b^2}\ln\left|\frac{ax+b}{x}\right| - \frac{1}{bx}$
21. $\int \frac{dx}{x^2(ax+b)^2} = \frac{2a}{b^3}\ln\left|\frac{ax+b}{x}\right| - \frac{2ax+b}{b^2x(ax+b)}$
22. $\int \frac{dx}{x(x^2+a^2)} = \frac{1}{2a^2}\ln\left(\frac{x^2}{x^2+a^2}\right)$
23. $\int \frac{dx}{x^2(x^2+a^2)} = \frac{-1}{a^2x} - \frac{1}{a^3}\tan^{-1}\left(\frac{x}{a}\right)$
24. $\int \frac{x^2}{(x^2+a^2)^2}\,dx = \frac{1}{2a}\tan^{-1}\left(\frac{x}{a}\right) - \frac{x}{2(x^2+a^2)}$
25. $\int \frac{dx}{x(x^2-a^2)} = \frac{1}{2a^2}\ln\left|\frac{x^2-a^2}{x^2}\right|$
26. $\int \frac{dx}{x^2(x^2-a^2)} = \frac{1}{a^2x} + \frac{1}{2a^3}\ln\left|\frac{x-a}{x+a}\right|$
27. $\int \frac{x^2}{(x^2-a^2)^2}\,dx = \frac{x}{2(a^2-x^2)} - \frac{1}{4a}\ln\left|\frac{x+a}{x-a}\right|$
28. $\int \frac{dx}{x^2+ax+b} =$

$$\begin{cases} \frac{2}{\sqrt{4b-a^2}}\tan^{-1}\left(\frac{2x+a}{\sqrt{4b-a^2}}\right) & \text{if } 4b > a^2 \\ \frac{-1}{x+a/2} & \text{if } 4b = a^2 \\ \frac{1}{\sqrt{a^2-4b}}\ln\left|\frac{2x+a-\sqrt{a^2-4b}}{2x+a+\sqrt{a^2-4b}}\right| & \text{if } 4b < a^2 \end{cases}$$

29. $\int \frac{x}{x^2+ax+b}\,dx = \frac{1}{2}\ln\left|x^2+ax+b\right| +$

$$\begin{cases} \frac{a}{\sqrt{4b-a^2}}\tan^{-1}\left(\frac{2x+a}{\sqrt{4b-a^2}}\right) & \text{if } 4b > a^2 \\ \frac{a}{2x+a} & \text{if } 4b = a^2 \\ \frac{-a}{2\sqrt{a^2-4b}}\ln\left|\frac{2x+a-\sqrt{a^2-4b}}{2x+a+\sqrt{a^2-4b}}\right| & \text{if } 4b < a^2 \end{cases}$$

30. $\int \frac{x^2}{x^2+ax+b}\,dx = x - \frac{a}{2}\ln\left|x^2+ax+b\right| +$

$$\begin{cases} \frac{a^2-2b}{\sqrt{4b-a^2}}\tan^{-1}\left(\frac{2x+a}{\sqrt{4b-a^2}}\right) & \text{if } 4b > a^2 \\ \frac{-a^2}{4x+2a} & \text{if } 4b = a^2 \\ \frac{a^2-2b}{2\sqrt{a^2-4b}}\ln\left|\frac{2x+a-\sqrt{a^2-4b}}{2x+a+\sqrt{a^2-4b}}\right| & \text{if } 4b < a^2 \end{cases}$$

31. $\int \frac{dx}{x^3+a^3} = \frac{1}{6a^2}\ln\left|\frac{x^2+2ax+a^2}{x^2-ax+a^2}\right| + \frac{1}{\sqrt{3}a^2}\tan^{-1}\left(\frac{2x-a}{\sqrt{3}a}\right)$

32. $\int \frac{x}{x^3+a^3}\,dx = \frac{1}{6a}\ln\left|\frac{x^2-ax+a^2}{x^2+2ax+a^2}\right| + \frac{1}{\sqrt{3}a}\tan^{-1}\left(\frac{2x-a}{\sqrt{3}a}\right)$

33. $\int \frac{dx}{x^4+a^4} = \frac{1}{4\sqrt{2}a^3}\ln\left|\frac{x^2+\sqrt{2}ax+a^2}{x^2-\sqrt{2}ax+a^2}\right| + \frac{1}{2\sqrt{2}a^3}\tan^{-1}\left(\frac{\sqrt{2}a}{a^2-x^2}\right)$

34. $\int \frac{x}{x^4+a^4}\,dx = \frac{-1}{2a^2}\tan^{-1}\left(\frac{a^2}{x^2}\right)$

35. $\int \frac{x^2}{x^4+a^4}\,dx = \frac{1}{4\sqrt{2}a}\ln\left|\frac{x^2-\sqrt{2}ax+a^2}{x^2+\sqrt{2}ax+a^2}\right| + \frac{1}{2\sqrt{2}a}\tan^{-1}\left(\frac{\sqrt{2}a}{a^2-x^2}\right)$

Square Roots

36. $\int \sqrt{ax+b}\,dx = \frac{2}{3a}(ax+b)^{3/2}$

37. $\int \frac{dx}{\sqrt{ax+b}} = \frac{2}{a}\sqrt{ax+b}$

38. $\int x\sqrt{ax+b}\,dx = \frac{6ax-4b}{15a^2}(ax+b)^{3/2}$

39. $\int \frac{x}{\sqrt{ax+b}}\,dx = \frac{2ax-4b}{3a^2}\sqrt{ax+b}$

40. $\int \frac{\sqrt{ax+b}}{x}\,dx = 2\sqrt{ax+b} + \begin{cases} \sqrt{b}\ln\left|\frac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}}\right| & \text{if } b>0 \\ -2\sqrt{-b}\tan^{-1}\left(\sqrt{\frac{ax+b}{-b}}\right) & \text{if } b<0 \end{cases}$

41. $\int \frac{dx}{x\sqrt{ax+b}} = \begin{cases} \frac{1}{\sqrt{b}}\ln\left|\frac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}}\right| & \text{if } b>0 \\ \frac{2}{\sqrt{-b}}\tan^{-1}\left(\sqrt{\frac{ax+b}{-b}}\right) & \text{if } b<0 \end{cases}$

42. $\int \sqrt{\frac{ax+b}{rx+s}}\,dx = \frac{2(as-br)}{r^2}\int \frac{u^2}{(u^2-a/c)^2}\,du$

where $u = \sqrt{\frac{ax+b}{rx+s}}$, see eqs. 24 & 27

43. $\int \frac{dx}{a\sqrt{x}+b} = \frac{2\sqrt{x}}{a} - \frac{2b}{a^2}\ln|a\sqrt{x}+b|$

44. $\int \frac{\sqrt{x}}{a\sqrt{x}+b}\,dx = \frac{x}{a} - \frac{2b\sqrt{x}}{a^2} + \frac{2b^2}{a^3}\ln|a\sqrt{x}+b|$

45. $\int \frac{x}{a\sqrt{x}+b}\,dx = \frac{2}{3a}x^{3/2} - \frac{bx}{a^2} + \frac{2b^2}{a^3}\sqrt{x} - \frac{2b^3}{a^4}\ln|a\sqrt{x}+b|$

46. $\int \sqrt{x+a\sqrt{x}+b}\,dx = \left(\frac{2}{3}x - \frac{a}{6}\sqrt{x} + \frac{2b}{3} - \frac{a^2}{4}\right)(x+a\sqrt{x}+b)^{3/2} - \left(ab - \frac{a^2}{4}\right)\ln\left|\sqrt{x}+\frac{a}{2}+\sqrt{x+a\sqrt{x}+b}\right|$

47. $\int \sqrt{x^2+a^2}\,dx = \frac{1}{2}x\sqrt{x^2+a^2} + \frac{a^2}{2}\ln\left|x+\sqrt{x^2+a^2}\right|$

48. $\int \sqrt{x^2-a^2}\,dx = \frac{1}{2}x\sqrt{x^2-a^2} - \frac{a^2}{2}\ln\left|x+\sqrt{x^2-a^2}\right|$

49. $\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}x\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left|\frac{x}{a}\right|$

50. $\int x^2\sqrt{x^2+a^2}\,dx = \frac{2x^2+a^2}{8}x\sqrt{x^2+a^2} - \frac{a^4}{8}\ln\left|x+\sqrt{x^2+a^2}\right|$

51. $\int x^2\sqrt{x^2-a^2}\,dx = \frac{2x^2-a^2}{8}x\sqrt{x^2-a^2} - \frac{a^4}{8}\ln\left|x+\sqrt{x^2-a^2}\right|$

52. $\int x^2\sqrt{a^2-x^2}\,dx =$

$\frac{2x^3-a^2x}{8}\sqrt{a^2-x^2}+\frac{a^4}{8}\sin^{-1}\left(\frac{x}{a}\right)$

53. $\int \frac{\sqrt{x^2+a^2}}{x}\,dx = \sqrt{x^2+a^2}-$

$a\ln\left|\frac{a+\sqrt{x^2+a^2}}{x}\right|$

54. $\int \frac{\sqrt{x^2-a^2}}{x}\,dx = \sqrt{x^2-a^2}-a\sec^{-1}\left(\frac{x}{a}\right)$

55. $\int \frac{\sqrt{a^2-x^2}}{x}\,dx = \sqrt{a^2-x^2}-$

$a\ln\left|\frac{a+\sqrt{a^2-x^2}}{x}\right|$

56. $\int \frac{\sqrt{x^2+a^2}}{x^2}\,dx = -\frac{\sqrt{x^2+a^2}}{x}+$

$\ln\left|x+\sqrt{x^2+a^2}\right|$

57. $\int \frac{\sqrt{x^2-a^2}}{x^2}\,dx = -\frac{\sqrt{x^2-a^2}}{x}+$

$\ln\left|x+\sqrt{x^2-a^2}\right|$

58. $\int \frac{\sqrt{a^2-x^2}}{x^2}\,dx = -\frac{\sqrt{a^2-x^2}}{x}-\sin^{-1}\left(\frac{x}{a}\right)$

59. $\int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left|x+\sqrt{x^2+a^2}\right|$

60. $\int \frac{dx}{\sqrt{x^2-a^2}} = \ln\left|x+\sqrt{x^2-a^2}\right|$

61. $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right)$

62. $\int \frac{x^2}{\sqrt{x^2+a^2}}\,dx = \frac{1}{2}x\sqrt{x^2+a^2}-$

$\frac{a^2}{2}\ln\left|x+\sqrt{x^2+a^2}\right|$

63. $\int \frac{x^2}{\sqrt{x^2-a^2}}\,dx = \frac{1}{2}x\sqrt{x^2-a^2}+$

$\frac{a^2}{2}\ln\left|x+\sqrt{x^2-a^2}\right|$

64. $\int \frac{x^2}{\sqrt{a^2-x^2}}\,dx = \frac{1}{2}x\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)$

65. $\int \frac{dx}{x\sqrt{x^2+a^2}} = \frac{-1}{a}\ln\left|\frac{a+\sqrt{x^2+a^2}}{x}\right|$

66. $\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right|$

67. $\int \frac{dx}{x\sqrt{a^2-x^2}} = \frac{-1}{a}\ln\left|\frac{a+\sqrt{a^2-x^2}}{x}\right|$

68. $\int \frac{dx}{x^2\sqrt{x^2+a^2}} = \frac{-\sqrt{x^2+a^2}}{a^2x}$

69. $\int \frac{dx}{x^2\sqrt{x^2-a^2}} = \frac{\sqrt{x^2-a^2}}{a^2x}$

70. $\int \frac{dx}{x^2\sqrt{a^2-x^2}} = \frac{-\sqrt{a^2-x^2}}{a^2x}$

71. $\int \frac{dx}{x+\sqrt{x^2+a^2}} = \frac{x}{2a^2}\sqrt{x^2+a^2}-$

$\frac{x^2}{2a^2}+\frac{1}{2}\ln(x+\sqrt{x^2+a^2})$

72. $\int \frac{dx}{x+\sqrt{x^2-a^2}} = \frac{-x}{2a^2}\sqrt{x^2-a^2}+$

$\frac{x^2}{2a^2}+\frac{1}{2}\ln(x+\sqrt{x^2-a^2})$

73. $\int \frac{dx}{x+\sqrt{a^2-x^2}} = \ln(x+\sqrt{a^2-x^2})-$

$\frac{1}{4}\ln|2x^2-a^2|+\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$

74. $\int \frac{dx}{a+\sqrt{a^2-x^2}} = \frac{\sqrt{a^2-x^2}}{x}+$

$\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)-\frac{a}{x}$

Natural Logarithms

75. $\int x^n\ln x\,dx = \frac{x^{n+1}\ln x}{n+1}-\frac{x^{n+1}}{(n+1)^2},\quad n\neq 1$

76. $\int \frac{\ln x}{x}\,dx = \frac{1}{2}(\ln x)^2$

77. $\int \frac{(\ln x)^m}{x}\,dx = \frac{(\ln x)^{m+1}}{m+1},\quad m\neq -1$

78. $\int \frac{dx}{x\ln x} = \ln|\ln x|$

79. $\int \frac{\ln x}{(x+a)^2}\,dx = \frac{1}{a}\left(\frac{x}{x+a}\ln x-\ln|x+a|\right)$

80. $\int (\ln x)^2\,dx = x[(\ln x)^2-2\ln x+2]$

81. $\int \ln(a\sqrt{x}+b)\,dx =$

$(x-\frac{b^2}{a^2})\ln(a\sqrt{x}+b)-\frac{x}{2}+\frac{b\sqrt{x}}{a}$

82. $\int \ln(x^2+ax+b)\,dx = -2x-a+$

$(x+\frac{a}{2})\ln(x^2+ax+b)+$

$$\begin{cases} \sqrt{4b-a^2}\tan^{-1}\left(\frac{2x}{\sqrt{4b-a^2}}\right) & \text{if } 4b>a^2 \\ 0 & \text{if } 4b=a^2 \\ \frac{1}{2}\sqrt{a^2-4b}\ln\left|\frac{2x+a+\sqrt{a^2-4b}}{2x+a-\sqrt{a^2-4b}}\right| & \text{if } 4b<a^2 \end{cases}$$

83. $\int \ln(x+\sqrt{x^2+a^2})\,dx = -\sqrt{x^2+a^2}+$

$x\ln(x+\sqrt{x^2+a^2})$

84. $\int \ln(x+\sqrt{x^2-a^2})\,dx = -\sqrt{x^2-a^2}+$

$x\ln(x+\sqrt{x^2-a^2})$

85. $\int \ln(x-\sqrt{x^2-a^2})\,dx = 2x\ln a+$

$\sqrt{x^2-a^2}-x\ln(x+\sqrt{x^2-a^2})$

86. $\int \ln(x+\sqrt{a^2-x^2})\,dx =$

$x\ln(x+\sqrt{a^2-x^2})-x-$

$a\ln\left|\frac{a+\sqrt{a^2-x^2}}{x}\right|$

87. $\int \ln(a+\sqrt{a^2-x^2})\,dx = x+$

$x\ln(a+\sqrt{a^2-x^2})-a\sin^{-1}\left(\frac{x}{a}\right)$

88. $\int \ln(x^3+a^3)\,dx = x\ln(x^3+a^3)-x+$

$\frac{a}{2}\ln\left|\frac{x^2+2ax+a^2}{x^2-ax+a^2}\right|+\sqrt{3}a\tan^{-1}\left(\frac{2x}{\sqrt{3}a}\right)$

Exponential Functions

89. $\int xe^{ax}\,dx = \frac{xe^{ax}}{a}-\frac{e^{ax}}{a^2}$

90. $\int x^2e^{ax}\,dx = \frac{x^2e^{ax}}{a}-\frac{2xe^{ax}}{a^2}+\frac{2e^{ax}}{a^3}$

91. $\int e^{a\sqrt{x}}\,dx = \frac{2}{a}\sqrt{x}e^{a\sqrt{x}}-\frac{2}{a^2}e^{a\sqrt{x}}$

92. $\int \frac{dx}{b+e^{ax}} = \frac{x}{b}-\frac{1}{ab}\ln|b+e^{ax}|$

93. $\int \frac{e^{ax}}{b+e^{ax}}\,dx = \frac{1}{a}\ln|b+e^{ax}|$

94. $\int \sqrt{e^{ax}+b}\,dx = \frac{2}{a}\sqrt{e^{ax}+b}+$

$$\begin{cases} \frac{\sqrt{b}}{a}\ln\left|\frac{\sqrt{e^{ax}+b}-\sqrt{b}}{\sqrt{e^{ax}+b}+\sqrt{b}}\right| & \text{if } b>0 \\ -\frac{2\sqrt{-b}}{a}\tan^{-1}\left(\frac{\sqrt{e^{ax}+b}}{\sqrt{-b}}\right) & \text{if } b<0 \end{cases}$$

95. $\int \sqrt{b-e^{ax}}\,dx = \frac{2}{a}\sqrt{b-e^{ax}}+$

$\frac{\sqrt{b}}{a}\ln\left|\frac{\sqrt{e^{ax}+b}-\sqrt{b}}{\sqrt{e^{ax}+b}+\sqrt{b}}\right|$

96. $\int \frac{dx}{\sqrt{b-e^{ax}}} = \frac{\sqrt{b}}{a}\ln\left|\frac{\sqrt{b-e^{ax}}-\sqrt{b}}{\sqrt{b-e^{ax}}+\sqrt{b}}\right|$

97. $\int \frac{dx}{\sqrt{e^{ax}+b}} =$

$$\begin{cases} \frac{\sqrt{b}}{a}\ln\left|\frac{\sqrt{e^{ax}+b}-\sqrt{b}}{\sqrt{e^{ax}+b}+\sqrt{b}}\right| & \text{if } b>0 \\ \frac{2\sqrt{-b}}{a}\tan^{-1}\left(\frac{\sqrt{e^{ax}+b}}{\sqrt{-b}}\right) & \text{if } b<0 \end{cases}$$

98. $\int e^{ax}\sin(bx)\,dx =$

$\frac{e^{ax}}{a^2+b^2}[a\sin(bx)-b\cos(bx)]$

99. $\int e^{ax}\cos(bx)\,dx =$

$\frac{e^{ax}}{a^2+b^2}[b\sin(bx)+a\cos(bx)]$

100. $\int xe^{ax}\sin(bx)\,dx = \frac{e^{ax}}{a^2+b^2}[ax\sin(bx) - bx\cos(bx) - \frac{a^2-b^2}{a^2+b^2}\sin(bx) + \frac{2ab}{a^2+b^2}\cos(bx)]$

101. $\int xe^{ax}\cos(bx)\,dx = \frac{e^{ax}}{a^2+b^2}[bx\sin(bx) + ax\cos(bx) - \frac{a^2-b^2}{a^2+b^2}\cos(bx) - \frac{2ab}{a^2+b^2}\sin(bx)]$

Trigonometric Functions

102. $\int \sin^2(x)\,dx = \frac{x}{2} - \frac{\sin(2x)}{4}$

103. $\int \sin^3(x)\,dx = -\sin^2(x)\cos(x) - \frac{2}{3}\cos^3(x)$

104. $\int \cos^2(x)\,dx = \frac{x}{2} + \frac{\sin(2x)}{4}$

105. $\int \cos^3(x)\,dx = \sin(x)\cos^2(x) + \frac{2}{3}\sin^3(x)$

106. $\int \sin^2(x)\cos^2(x)\,dx = \frac{x}{16} - \frac{\sin(4x)}{32}$

107. $\int x\sin(x)\,dx = \sin(x) - x\cos(x)$

108. $\int x^2\sin(x)\,dx = 2\cos(x) + 2x\sin(x) - x^2\cos(x)$

109. $\int x\sin^2(x)\,dx = \frac{x^2}{4} - \frac{x\sin(2x)}{4} - \frac{\cos(2x)}{8}$

110. $\int x^2\sin^2(x)\,dx = \frac{x^3}{6} - \frac{x^2\sin(2x)}{4} - \frac{x\cos(2x)}{4} + \frac{\sin(2x)}{8}$

111. $\int x\cos(x)\,dx = \cos(x) + x\sin(x)$

112. $\int x^2\cos(x)\,dx = -2\sin(x) + 2x\cos(x) + x^2\sin(x)$

113. $\int x\cos^2(x)\,dx = \frac{x^2}{4} + \frac{x\sin(2x)}{4} + \frac{\cos(2x)}{8}$

114. $\int x^2\cos^2(x)\,dx = \frac{x^3}{6} + \frac{x^2\sin(2x)}{4} + \frac{x\cos(2x)}{4} - \frac{\sin(2x)}{8}$

115. $\int \frac{dx}{\sin(x)} = \ln\left|\frac{1-\cos(x)}{\sin(x)}\right|$

116. $\int \frac{dx}{\cos(x)} = \ln\left|\frac{1+\sin(x)}{\cos(x)}\right|$

117. $\int \frac{dx}{\sin^2(x)} = -\cot(x)$

118. $\int \frac{dx}{\cos^2(x)} = \tan(x)$

119. $\int \frac{dx}{\sin^3(x)} = \frac{-\cos^2(x)}{2\sin(x)} + \frac{1}{2}\ln\left|\frac{1-\cos(x)}{\sin(x)}\right|$

120. $\int \frac{dx}{\cos^3(x)} = \frac{\sin^2(x)}{2\cos(x)} + \frac{1}{2}\ln\left|\frac{1+\sin(x)}{\cos(x)}\right|$

121. $\int \frac{dx}{\sin^2(x)\cos(x)} = \ln\left|\frac{1+\sin(x)}{\cos(x)}\right| - \frac{1}{\sin(x)}$

122. $\int \frac{dx}{\sin(x)\cos^2(x)} = \ln\left|\frac{1-\cos(x)}{\sin(x)}\right| + \frac{1}{\cos(x)}$

123. $\int \sin(x)\sin(x+a)\,dx = \frac{1}{2}x\cos(a) - \frac{1}{4}\sin(2x+a)$

124. $\int \sin(ax)\sin(bx)\,dx = \frac{\sin([a-b]x)}{2(a-b)} - \frac{\sin([a+b]x)}{2(a+b)}$

125. $\int \cos(ax)\cos(bx)\,dx = \frac{\sin([a-b]x)}{2(a-b)} + \frac{\sin([a+b]x)}{2(a+b)}$

126. $\int \sin(ax)\cos(bx)\,dx = \frac{-\cos([a-b]x)}{2(a-b)} - \frac{\cos([a+b]x)}{2(a+b)}$

127. $\int \frac{dx}{1\pm\sin(x)} = \mp\tan\left(\frac{x}{4} \mp \frac{x}{2}\right)$

128. $\int \frac{dx}{1\pm\cos(x)} = \mp\tan\left(\frac{x}{4}\mp\left(\frac{2x+\pi}{4}\right)\right)$

129. $\int \frac{x}{1\pm\sin(x)}\,dx = x\tan\left(\frac{x}{4}\mp\frac{\pi}{2}\right)\mp 2\ln\left|\sin\left(\frac{x}{4}\pm\frac{\pi}{2}\right)\right|$

130. $\int \frac{x}{1\pm\cos(x)}\,dx = (x+\frac{\pi}{2})\tan\left(\frac{x}{4}\mp\left(\frac{2x+\pi}{4}\right)\right)\mp 2\ln\left|\sin\left(\frac{x}{4}\pm\left(\frac{2x+\pi}{4}\right)\right)\right|$

131. $\int \frac{dx}{a+\sin(x)} =$

$$\begin{cases} \frac{2}{\sqrt{a^2-1}}\tan^{-1}\left[\frac{\sqrt{a^2-1}}{|a-1|}\tan\left(\frac{2x+\pi}{4}\right)\right] & \text{if } |a|>1 \\ \frac{1}{\sqrt{1-a^2}}\ln\left|\frac{a\tan(x/2)+1-\sqrt{1-a^2}}{a\tan(x/2)+1+\sqrt{1-a^2}}\right| & \text{if } |a|<1 \end{cases}$$

132. $\int \frac{dx}{a+\cos(x)} =$

$$\begin{cases} \frac{2}{\sqrt{a^2-1}}\tan^{-1}\left[\frac{|a-1|}{\sqrt{a^2-1}}\tan\left(\frac{x}{2}\right)\right] & \text{if } |a|>1 \\ \frac{1}{\sqrt{1-a^2}}\ln\left|\frac{\tan(x/2)+\frac{\sqrt{1-a^2}}{1+a}}{\tan(x/2)-\frac{\sqrt{1-a^2}}{1+a}}\right| & \text{if } |a|<1 \end{cases}$$

133. $\int \frac{dx}{a\sin(x)+\cos(x)} = \frac{1}{\sqrt{a^2+1}}\ln\left|\tan\left(\frac{x+\tan^{-1}\frac{1}{a}}{2}\right)\right|$

134. $\int \frac{\cos(x)}{a\sin(x)+\cos(x)}dx = \frac{x}{a^2+1}+\frac{a}{2(a^2+1)}\ln\left|a+\frac{a^2+1}{2}\sin(2x)\right|+\frac{a}{a^2+1}\ln\left|\frac{a\tan(x)+1}{a\tan(x)+a^2}\right|$

135. $\int \tan^2(x)\,dx = \tan(x)-x$

136. $\int \tan^3(x)\,dx = \frac{1}{2}\tan^2(x)+\ln|\cos(x)|$

137. $\int \cot^2(x)\,dx = -\cot(x)-x$

138. $\int \cot^3(x)\,dx = -\frac{1}{2}\cot^2(x)-\ln|\sin(x)|$

139. $\int \sin^{-1}(x)\,dx = x\sin^{-1}(x)+\sqrt{1-x^2}$

140. $\int (\sin^{-1}(x))^2\,dx = x(\sin^{-1}(x))^2 - 2x+2\sin^{-1}(x)\sqrt{1-x^2}$

141. $\int \sin^{-1}\left(\frac{1}{x}\right)\,dx = x\sin^{-1}\left(\frac{1}{x}\right)+\ln(x+\sqrt{x^2-1})$

142. $\int \cos^{-1}(x)\,dx = x\cos^{-1}(x)-\sqrt{1-x^2}$

143. $\int (\cos^{-1}(x))^2\,dx = x(\cos^{-1}(x))^2 - 2x-2\cos^{-1}(x)\sqrt{1-x^2}$

144. $\int \cos^{-1}\left(\frac{1}{x}\right)\,dx = x\cos^{-1}\left(\frac{1}{x}\right)-\ln(x+\sqrt{x^2-1})$

145. $\int (\sin^{-1}(x))(\cos^{-1}(x))\,dx = x\sin^{-1}(x)\cos^{-1}(x)+2x+\sqrt{1-x^2}(\cos^{-1}(x)-\sin^{-1}(x))$

146. $\int \sin(a\sin^{-1}(x))dx = \frac{a\sqrt{1-x^2}\cos(a\sin^{-1}(x))+x\sin(a\sin^{-1}(x))}{1-a^2}$

147. $\int \cos(a\cos^{-1}(x))dx = \frac{a\sqrt{1-x^2}\sin(a\cos^{-1}(x))+x\cos(a\cos^{-1}(x))}{1-a^2}$

148. $\int \tan^{-1}(x)\,dx = x\tan^{-1}(x) - \frac{1}{2}\ln(x^2+1)$

149. $\int \tan(a+\tan^{-1}(x))\,dx = x\cot(a) - \frac{1}{\sin^2(a)}\ln(x\sin(a)-\cos(a))$

150. $\int x\sin^{-1}(x)\,dx = \frac{(2x^2-1)\sin^{-1}x+x\sqrt{1-x^2}}{4}$

151. $\int x\cos^{-1}(x)\,dx = \frac{(2x^2-1)\cos^{-1}x-x\sqrt{1-x^2}}{4}$

152. $\int x\tan^{-1}(x)\,dx = \frac{(x^2+1)\tan^{-1}(x)-x}{2}$

153. $\int \tan^{-1}\left(\frac{ax+b}{x+c}\right)\,dx = \left(x+\frac{ab+c}{a^2+1}\right)\tan^{-1}\left(\frac{ax+b}{x+c}\right) + \frac{b-ac}{2a^2+2}\ln((ax+b)^2+(x+c)^2)$

Hyperbolic Functions

154. $\sinh(x) = \frac{e^x-e^{-x}}{2}$

155. $\cosh(x) = \frac{e^x+e^{-x}}{2}$

156. $\tanh(x) = \frac{e^x-e^{-x}}{e^x+e^{-x}}$

157. $\cosh^2(x) - \sinh^2(x) = 1$

158. $\sinh^{-1} = \ln(x+\sqrt{x^2+1})$

159. $\cosh^{-1} = \ln(x+\sqrt{x^2-1})$

160. $\tanh^{-1} = \ln\left(\frac{1+x}{1-x}\right)$

161. $\int \sinh(x)\,dx = \cosh(x)$

162. $\int \cosh(x)\,dx = \sinh(x)$

163. $\int \tanh(x)\,dx = \ln(e^x+e^{-x})$

164. $\int \frac{dx}{\sinh(x)} = \ln\left|\frac{e^x-1}{e^x+1}\right|$

165. $\int \frac{dx}{\cosh(x)} = 2\tan^{-1}(e^x)$

166. $\int \frac{dx}{\sinh^2(x)} = -\coth(x) = \frac{-1}{\tanh(x)}$

167. $\int \frac{dx}{\cosh^2(x)} = \tanh(x)$

168. $\int \tanh^2(x)\,dx = x - \tanh(x)$

169. $\int x\sinh(x)\,dx = x\cosh(x) - \sinh(x)$

170. $\int x\cosh(x)\,dx = x\sinh(x) - \cosh(x)$

171. $\int \sinh^{-1}(x)\,dx = x\sinh^{-1}(x) - \sqrt{x^2+1}$

172. $\int \cosh^{-1}(x)\,dx = x\cosh^{-1}(x) - \sqrt{x^2-1}$

173. $\int \tanh^{-1}(x)\,dx = \frac{1}{2}\ln(1-x^2) + x\tanh^{-1}(x)$

174. $\int x\sinh^{-1}(x)\,dx = \frac{2x^2+1}{4}\sinh^{-1}(x) - \frac{1}{4}\sqrt{x^2+1}$

175. $\int x\cosh^{-1}(x)\,dx = \frac{2x^2+1}{4}\cosh^{-1}(x) - \frac{1}{4}\sqrt{x^2-1}$

176. $\int x\tanh^{-1}(x)\,dx = \frac{1}{2}(x^2-1)\tanh^{-1}(x) + \frac{x}{2}$

177. $\int (\sinh^{-1}(x))^2\,dx = 2x + x(\sinh^{-1}(x))^2 - 2\sinh^{-1}(x)\sqrt{x^2+1}$

178. $\int (\cosh^{-1}(x))^2\,dx = 2x + x(\cosh^{-1}(x))^2 - 2\cosh^{-1}(x)\sqrt{x^2-1}$

179. $\int \sinh^{-1}\left(\frac{1}{x}\right)\,dx = x\sinh^{-1}\left(\frac{1}{x}\right) + \sinh^{-1}(x)$

180. $\int \cosh^{-1}\left(\frac{1}{x}\right)\,dx = x\cosh^{-1}\left(\frac{1}{x}\right) + \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$

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