Calculus 141, section 8.7b Improper Integrals

notes by Tim Pilachowski

Improper integrals that have a numeric value are said to be *convergent*. The rest are said to be *divergent*.

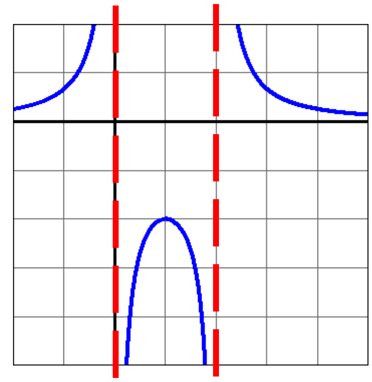
basic method: $\int_a^{\infty} f(x)\,dx = \lim_{b \to \infty} \int_a^b f(x)\,dx$, with associated versions for integrals involving $-\infty$ and denominators approaching 0.

Example A: Does $\int_e^{\infty} \frac{dx}{x(\ln x)^2}$ converge? *Answer*: converges to 1

Example B: Find the volume of $y = \frac{1}{x}$ rotated around x-axis from 1 to ∞. *Answer*: converges to π

A paradox: The area under the curve, in two dimensions, is infinite, but the volume of the three-dimensional figure is bound by an upper limit of π! However, the surface area of the same figure is also infinite.

Example C: Does $\int_1^2 \frac{2}{x^2 - 2x}\,dx$ converge? *Answer*: diverges



Example D: Does $\int_0^\infty x\,3^{-x}\,dx$ converge? *Answer*: converges to $\left(\frac{1}{\ln 3}\right)^2$