Stat 400, chapter 2, Probability, Permutations and Combinations - SOLUTIONS

supplemental handout prepared by Tim Pilachowski

Examples A. through F. have answers on the supplement, with explanations.

Example G. In a game of bridge, using a standard deck of fifty-two playing cards, each of four players is dealt thirteen cards. Is this an example of a combination or permutation? How many possible bridge hands are there?

combination – The cards were shuffled so that they would not be in any particular order when dealt.

$$\binom{52}{13} = \frac{52!}{13!(52-13)!} = \frac{52*51*50*49*48*47*46*45*44*43*42*41*40*39!}{13!\,39!} = 635{,}013{,}559{,}600$$

Example H. a) Odysseus can name every player on his baseball team's roster. Is this an example of a combination or permutation? b) As the club's manager, Persephone decides which player will bat when in the lineup for each game. Is this an example of a combination or permutation? c) If the team roster has 25 members, how many ways can Persephone set up the starting lineup of nine batters?

a) combination – He can name the team in alphabetical order, by position, by size, etc. A specific order is not needed. b) permutation – Order is important. The team will want a different batter to be first up than one who bats in the clean-up position.

c) $P_{9,25} = \frac{25!}{(25-9)!} = \frac{25*24*23*22*21*20*19*18*17*16!}{16!} = 741{,}354{,}768{,}000$

Example I-a. One hundred names are put into a hat. Is this an example of a combination or permutation?

combination – The purpose of putting names in a hat is specifically because they are not in any order, and names will be chosen randomly.

Example I-b. One hundred names are put into a hat. Names will be drawn for prizes. The top prize is $500, the next prize is $300, and the third prize is $100. Is this an example of a combination or permutation? How many ways can the prizes be awarded?

permutation – Order matters, especially to the prize winners.

$$P_{3,100} = \frac{100!}{(100-3)!} = \frac{100*99*98*97!}{97!} = 970{,}200$$

Example I-c. One hundred names are put into a hat. Names will be drawn for three prizes of $300 each. Is this an example of a combination or permutation? How many ways can the prizes be awarded?

combination – Order does not matter. Prize winners are not worried about whether their names are drawn first, second or third, as long as they're drawn.

$$\binom{100}{3} = \frac{100!}{3!(100-3)!} = \frac{100*99*98*97!}{3!\,97!} = 161{,}700$$

Answers to Examples J and K are on the next page.

Example J. For the Powerball lottery, “Every Wednesday and Saturday night, five white balls from 1 to 59 and one red Powerball from 1 to 39 will be drawn. You win a prize by matching some or all of the numbers drawn.” (source: mdlottery.com). Are the choices for a Powerball ticket a combination or a permutation? How many ways can a player pick the numbers for a Powerball ticket?

both combination *and* permutation – The five white balls do not have to be drawn in any particular order. However, the single white ball is completely separate from the red ones.

$$\binom{59}{5} * P_{1,39} = \frac{59!}{5!(59-5)!} * 39 = \frac{59*58*57*56*55*54!}{5*4*3*2*1*54!} * 39 = 195{,}249{,}054$$

Example K-a. A club is electing its four officers. Is this an example of a combination or permutation? If the club has fifty members, how many different ways can slates of officers be formed?

permutation – Order matters since each officer has a different job.

$$P_{4,50} = \frac{50!}{(50-4)!} = \frac{50*49*48*47*16!}{16!} = 5{,}527{,}200$$

Example K-b. A club is forming a committee of four to examine and make suggestions to revise the bylaws. Is this an example of a combination or permutation? If the club has fifty members, how many different ways can the Bylaw Review Committee be formed?

combination – Order does not matter since the committee members all contribute and jobs have not been assigned.

$$\binom{50}{4} = \frac{P_{4,50}}{4!} = \frac{50!}{4!(50-4)!} = \frac{50*49*48*47*46!}{4*3*2*1*46!} = 230{,}300$$

Example K-c. A club is forming a committee to plan its annual Equinox Dance. One person will serve as Chair of the committee, and there will be three other members. Is this an example of a combination or permutation? If the club has fifty members, how many different ways can the Equinox Dance Committee be formed?

both combination and permutation – Order matters in the selection of the Chair, but not in the selection of the other committee members. 921,200

$$P_{1,50} * \binom{49}{3} = 50 * \frac{49!}{3!(49-3)!} = 50 * \frac{49*48*47*46!}{3*2*1*46!} = 921{,}200$$

Example K-d. A club is forming a committee to plan its annual Holiday Dance. One person will serve as Chair of the committee, another will be Vice-Chair and there will be two other members. Is this an example of a combination or permutation? If the club has fifty members, how many different ways can the Holiday Dance Committee be formed?

both combination and permutation – Order matters for choosing Chair and Vice-Chair, however, the choice of committee members is a combination. 2,763,600

$$P_{2,50} * \binom{48}{2} = 50 * 49 * \frac{48!}{2!(48-2)!} = 50 * 49 * \frac{48*47*46!}{2*1*46!} = 2{,}763{,}600$$