Stat 400, chapter 2, Probability, Permutations and Combinations
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An arrangement in which the order of objects or events makes a difference, e.g. RYB ≠ RBY, is called a *permutation*. When the order of objects or events does not make a difference, e.g. RYB = RBY = BYR, etc, we have a *combination*.

Answer the questions in the Examples below first yourself, first. Only look at the worked-out answers after you've tried them for yourself.

Example A. a) In Arithmetic, addition is both commutative and associative, that is numbers can be rearranged and regrouped in any way and the answer will still be the same. Is addition more like a combination or permutation? b) In Arithmetic, division is neither commutative nor associative. It makes a difference whether one is calculating "p divided by q" or "q divided by p". Is division more like a combination or permutation?

Example B. a) The cook at Pappy's Restaurant makes a pizza by first rolling out and stretching the dough for the crust, then spreading out the sauce, then adding cheese and toppings. Is making a pizza more like a combination or permutation? b) The cook at Bel-Loc Diner makes sausage stew by throwing all of the ingredients into a pot and letting it simmer for four hours. Is making stew more like a combination or permutation?

Example C. a) Harold Hill puts his knick knacks on a shelf according to whatever whim hits him at the moment, and is likely to move them around periodically. Is Harold's method an example of a combination or permutation? b) Marian puts books on the shelf in the town's library, arranging them by using their call numbers. Is Marian's method an example of a combination or permutation?

Example D. In the Maryland Lottery's Pick 4 game, players have a choice of the type of bet. "Straight – Your number must match the winning number in exact order. Box – Your number can match the winning number in any order." (source: mdlottery.com) a) Is playing "Straight" an example of a combination or permutation? b) Is playing "Box" an example of a combination or permutation?

Example E. a) Twenty runners stand on the starting line at the beginning of a marathon. Is this an example of a combination or a permutation? b) Ribbons are awarded to first-, second-, and third-place finishers. Is this an example of a combination or a permutation?

Example F. A combination lock requires the user to know a set of three one-digit numbers which must be entered in a particular way. For example, if the correct combination is "1-3-5", then entering "5-1-3" or "3-5-1" won't open the lock. Is a "combination lock" an example of a combination or a permutation?

Example G. In a game of bridge, using a standard deck of fifty-two playing cards, each of four players is dealt thirteen cards. Is this an example of a combination or permutation? How many possible bridge hands are there?

Example H. a) Odysseus can name every player on his baseball team's roster. Is this an example of a combination or permutation? b) As the club's manager, Persephone decides which player will bat when in the lineup for each game. Is this an example of a combination or permutation? c) If the team roster has 25 members, how many ways can Persephone set up the starting lineup of nine batters?

Example I-a. One hundred names are put into a hat. Is this an example of a combination or permutation?

Example I-b. One hundred names are put into a hat. Names will be drawn for prizes. The top prize is $500, the next prize is $300, and the third prize is $100. Is this an example of a combination or permutation? How many ways can the prizes be awarded?

Example I-c. One hundred names are put into a hat. Names will be drawn for three prizes of $300 each. Is this an example of a combination or permutation? How many ways can the prizes be awarded?

Example J. For the Powerball lottery, "Every Wednesday and Saturday night, five white balls from 1 to 59 and one red Powerball from 1 to 39 will be drawn. You win a prize by matching some or all of the numbers drawn." (source: mdlottery.com). Are the choices for a Powerball ticket a combination or a permutation? How many ways can a player pick the numbers for a Powerball ticket?

Example K-a. A club is electing its four officers. Is this an example of a combination or permutation? If the club has fifty members, how many different ways can slates of officers be formed?

Example K-b. A club is forming a committee of four to examine and make suggestions to revise the bylaws. Is this an example of a combination or permutation? If the club has fifty members, how many different ways can the Bylaw Review Committee be formed?

Example K-c. A club is forming a committee to plan its annual Equinox Dance. One person will serve as Chair of the committee, and there will be three other members. Is this an example of a combination or permutation? If the club has fifty members, how many different ways can the Equinox Dance Committee be formed?

Example K-d. A club is forming a committee to plan its annual Holiday Dance. One person will serve as Chair of the committee, another will be Vice-Chair and there will be two other members. Is this an example of a combination or permutation? If the club has fifty members, how many different ways can the Holiday Dance Committee be formed?

A. a) combination –The order the numbers are written in does not matter. b) permutation – In division, it makes a difference which number is written first.

B. a) permutation –The crust **must** be on the bottom, to hold the other ingredients. Order makes a difference. b) combination – The carrots can go into the pot before or after the potatoes. Order doesn't matter, as long as all of the ingredients are included.

C. a) combination –Harold doesn't arrange his knick knacks according to any particular order, just three on this shelf, four over there, etc. b) permutation –Order is important. The call number system is used so people can find what the are looking for.

D. a) permutation –Order is important. If "1-2-3-4" is drawn, then only tickets with "1-2-3-4" are winners. b) combination – Order doesn't matter. If "1-2-3-4" is drawn, then tickets with "1-2-3-4", "1-2-4-3", "1-3-2-4", etc., are all winners.

E. a) combination - They have twenty-six miles to run. There is no advantage to being on the right or the left. Order doesn't matter. b) permutation – Order of finishing matters. The first-place runner wants to get the first-place ribbon.

F. permutation – The order of the numbers makes a difference. Calling it a "combination lock" is a mathematical misnomer.

G. combination – The cards were shuffled so that they would not be in any particular order when dealt; 635,013,559,600

H. a) combination – He can name the team in alphabetical order, by position, by size, etc. A specific order is not needed. b) permutation – Order is important. The team will want a different batter to be first up than one who bats in the clean-up position. c) 741,354,768,000

I. a) combination – The purpose of putting names in a hat is specifically because they are not in any order, and names will be chosen randomly. b) permutation – Order matters, especially to the prize winners. 970,200 c) combination – Order does not matter. Prize winners are not worried about whether their names are drawn first, second or third, as long as they're drawn. 161,700

J. both combination _and_ permutation – The five white balls do not have to be drawn in any particular order. However, the single white ball is completely separate from the red ones. 195,249,054

K. a) 5,527,200 b) 230,300 c) 921,200 d) 2,763,600