

Stat 400, chapter 2, Probability, Conditional Probability and Bayes' Theorem – Solutions
 supplemental handout prepared by Tim Pilachowski

Example 2: The Gallup organization conducted 10 separate surveys conducted from January through May 2009. At the time of the report, Gallup had found an average of 35% of Americans considering themselves Democratic, 37% independent and 28% Republican. Within those affiliations, the following percentages identified themselves as Conservative, Moderate or Liberal.

	Democrat (event D)	Independent (event I)	Republican (event R)
Conservative (event C)	22%	35%	73%
Moderate (event M)	40%	45%	24%
Liberal (event L)	38%	20%	3%

<http://www.gallup.com/poll/120857/conservatives-single-largest-ideological-group.aspx>

Results are based on aggregated Gallup Poll surveys of approximately 1,000 national adults, aged 18 and older, interviewed by telephone. Sample sizes for the annual compilations range from approximately 10,000 to approximately 40,000. For these results, one can say with 95% confidence that the maximum margin of sampling error is ± 1 percentage point.

This table gives conditional probabilities. $P(\text{Conservative} | \text{Independent}) = 35\%$. $P(\text{Liberal} | \text{Republican}) = 3\%$.

2-01. Determine $P(D)$, $P(I)$ and $P(R)$.

from the paragraph: $P(D) = 0.35$, $P(I) = 0.37$ and $P(R) = 0.28$

2-02. Determine $P(D^c)$, $P(I^c)$ and $P(R^c)$.

complement rule: $P(D^c) = 1 - 0.35 = 0.65$, $P(I^c) = 1 - 0.37 = 0.63$ and $P(R^c) = 1 - 0.28 = 0.72$

2-03a. Determine $P(M | D)$. b. Write a verbal description of what $P(M | D)$ means.

from the table: $P(M | D) = 0.4$; $P(M | D)$ is the probability that a person is Moderate given that he/she is a Democrat, or $P(M | D)$ is the probability that a Democrat is a Moderate.

2-04a. Write a verbal description of $D \cap M$. b. Calculate $P(D \cap M)$.

A person is both a Democrat and Moderate.; $P(D \cap M) = P(D) * P(M | D) = 0.35 * 0.4 = 0.14$

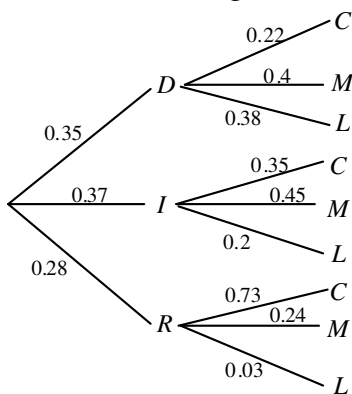
2-05a. Determine $P(M^c | R)$. b. Write a verbal description of what $P(M^c | R)$ means.

$M^c =$ not a Moderate = either Conservative or Liberal, from the table: $P(M^c | R) = 0.73 + 0.03 = 0.76$

2-06a. Write a verbal description of $R \cap M^c$. b. Calculate $P(R \cap M^c)$.

A person is a Republican and not a Moderate or A person is a Republican and either a Conservative or a Liberal.; $P(R \cap M^c) = P(R) * P(M^c | R) = 0.28 * 0.76 = 0.2128$

2-07. Draw a tree diagram to illustrate the events and probabilities for this two-stage experiment.



	Democrat (event D)	Independent (event I)	Republican (event R)
Conservative (event C)	0.077	0.1295	0.2044
Moderate (event M)	0.14	0.1665	0.0672
Liberal (event L)	0.133	0.074	0.0084

2-08. Calculate $P(M)$.

$$P(M) = P(D \cap M) + P(I \cap M) + P(R \cap M) = 0.35 * 0.4 + 0.37 * 0.45 + 0.28 * 0.24 = 0.3737$$

2-09a. Write a verbal description of $D \cup M$. b. Calculate $P(D \cup M)$.

A person is either a Democrat or a Moderate or both.; addition formula using given information with answers 2-08 and 2-04 above: $P(D \cup M) = P(D) + P(M) - P(D \cap M) = 0.35 + 0.3737 - 0.14 = 0.5837$

2-10a. Use Bayes' Theorem to calculate $P(D | M)$. b. Write a verbal description of what $P(D | M)$ means.

$$P(D | M) = \frac{P(D \cap M)}{P(M)} = \frac{0.14}{0.3737} \approx 0.3746; \text{ probability that a person who is a Moderate is also a Democrat}$$

2-11. Are D and M independent events? How do you know?

They are not independent since $P(D | M) \neq P(D)$.

2-12. Calculate $P(M^c)$.

$$P(M^c) = 1 - P(M) = 1 - 0.3737 = 0.6263$$

2-13a. Write a verbal description of $R \cup M^c$. b. Calculate $P(R \cup M^c)$.

A person is either a Republican or not a Moderate. *or* A person is either a Republican or a Conservative or a Liberal.; addition formula using given information with answers 2-12 and 2-06 above:

$$P(R \cup M^c) = P(R) + P(M^c) - P(R \cap M^c) = 0.28 + 0.6263 - 0.2128 = 0.6935$$

2-14a. Use Bayes' Theorem to calculate $P(R | M^c)$. b. Write a verbal description of what $P(R | M^c)$ means.

$$P(R | M^c) = \frac{P(R \cap M^c)}{P(M^c)} = \frac{0.2128}{0.6263} \approx 0.3398; \text{ probability that a person who is not a Moderate is a Republican}$$

2-15. Are R and M^c independent events? How do you know?

They are not independent since $P(R | M^c) \neq P(R)$.

Example 3: According to U.S. Department of Labor employment reports, in 2010, 39.3% of jobs were classified as Management, professional, and related occupations (Mgt), 14.5% were Service occupations (Svc), 23.2% were Sales and office occupations (SO), 9.9% were Natural resources, construction, and maintenance occupations (NCM), and 13.1% were Production, transportation, and material moving occupations (PTM). Among Mgt, 48.6% were held by male workers, and 51.4% were held by female workers. The other categories were: Svc, 50.6% male, 49.4% female; SO, 38.3% male, 61.7% female; NCM, 95.9% male, 4.1% female; PTM, 80.2% male, 19.8% female.

source: United States Department of Labor, Bureau of Labor Statistics report "Median weekly earnings of full-time wage and salary workers by detailed occupation and sex" covering the year 2010. <ftp://ftp.bls.gov/pub/special.requests/lf/aat39.txt>

Let A = Management/professional, B = Service, C = Sales/office, D = Natural resources/construction/maintenance, E = Production/transportation, M = male and F = female.

3-01a. Determine the probability that a person was not in a Management/Professional position. b. Determine the probability that a person was not in a Sales/Office position.

$$\text{complement rule: } P(A^c) = 1 - 0.393 = 0.607; P(C^c) = 1 - 0.232 = 0.768$$

3-02. Determine the probability that a person was male given that his job was classified as a service occupation. from the information provided: $P(M | B) = 0.506$

3-03. Calculate the probability that an individual was a man working in a service occupation.

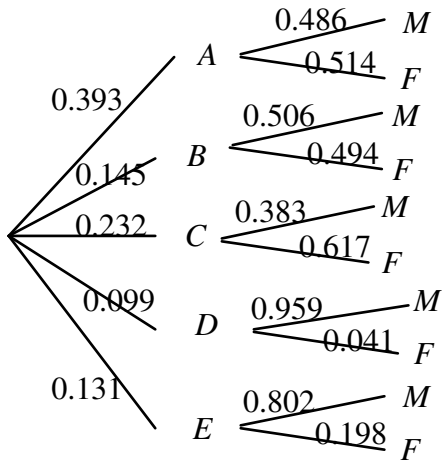
$$P(B \cap M) = P(B) * P(M | B) = 0.145 * 0.506 \approx 0.073$$

3-04. Determine the proportion of natural resources/construction/maintenance workers that were female. from the information provided: $P(F | D) = 0.041$

3-05. Determine the probability that a person was a woman employed as a natural resources/construction/ maintenance worker.

$$P(D \cap F) = P(D) * P(F | D) = 0.099 * 0.041 \approx 0.004$$

3-06. Draw a tree diagram to illustrate the events and probabilities given above.



	Male (event M)	Female (event F)
Mgt (event A)	0.190998	0.202002
Svc (event B)	0.07337	0.07163
SO (event C)	0.088856	0.143144
NCM (event D)	0.094941	0.004059
PTM (event E)	0.105062	0.025938

3-07a. Calculate the proportion of the 2010 U.S. workforce that was male. b. Calculate the proportion of the 2010 U.S. workforce that was female.

$$P(M) = P(A \cap M) + P(B \cap M) + P(C \cap M) + P(D \cap M) + P(E \cap M)$$

$$= 0.393 * 0.486 + 0.145 * 0.506 + 0.232 * 0.383 + 0.099 * 0.959 + 0.131 * 0.802 \approx 0.553$$

$$P(F) = P(M^c) = P(S) - P(M) = 1 - P(M) \approx 1 - 0.553 \approx 0.447$$

3-08. Calculate the probability that an individual is either a man or working in a service occupation. addition formula using given information with answers 3-07 and 3-05 above

$$P(B \cup M) = P(B) + P(M) - P(B \cap M) \approx 0.145 + 0.553 - 0.073 \approx 0.625$$

3-09. Determine the probability that a person was either female or was employed as a natural resources/construction/ maintenance worker.

addition formula using given information with answers 3-07 and 3-03 above

$$P(D \cup F) = P(D) + P(F) - P(D \cap F) \approx 0.099 + 0.447 - 0.004 \approx 0.542$$

3-10a. Calculate $P(D | F)$. b. Write a verbal description of what $P(D | F)$ means. c. Are D and F independent events? How do you know?

$$P(D | F) = \frac{P(D \cap F)}{P(F)} = \frac{0.004}{0.447} \approx 0.009; \text{ About 0.9\% (almost 1\%) of employed women work in Natural}$$

Resources, Construction or Maintenance occupations.; They are not independent since $P(D | F) \neq P(D)$. Note that if we had rounded off to two decimal places we would have reached an incorrect conclusion.

3-11a. Among the male members of the U.S. workforce, what proportion were in service occupations? b. Are being male and working in a service occupation independent events? How do you know?

$$P(B | M) = \frac{P(B \cap M)}{P(M)} \approx \frac{0.073}{0.553} \approx 0.132; \text{ About 13\% of employed men work in Service occupations.; They}$$

are not independent since $P(B | M) \neq P(B)$.