March 9, 2012 Stat 400 – EXAM 1 (2.1 – 4.2) [Pilachowski] **SOLUTIONS**

1.a. (4 points) Since values are equally likely this is a symmetric probability distribution, and the mean will be 2.

Or, $E(X)=(0)\left(\frac{1}{5}\right)+(1)\left(\frac{1}{5}\right)+(2)\left(\frac{1}{5}\right)+(3)\left(\frac{1}{5}\right)+(4)\left(\frac{1}{5}\right)=\left(\frac{1}{5}\right)(0+1+2+3+4)=\left(\frac{1}{5}\right)(10)=\frac{10}{4}=2$

$$E(X^2)=(0^2)\left(\frac{1}{5}\right)+(1^2)\left(\frac{1}{5}\right)+(2^2)\left(\frac{1}{5}\right)+(3^2)\left(\frac{1}{5}\right)+(4^2)\left(\frac{1}{5}\right)=\left(\frac{1}{5}\right)(0^2+1^2+2^2+3^2+4^2)=\frac{30}{5}=6$$

$$V(X)=E(X^2)-[E(X)]^2=6-(2)^2=2$$

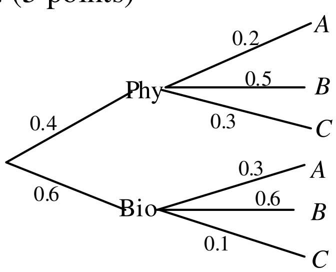
1.b. (4 points) $F(x)=\begin{cases}0 & x<0\\ 1/5 & 0\le x<1\\ 2/5 & 1\le x<2\\ 3/5 & 2\le x<3\\ 4/5 & 3\le x<4\\ 1 & 4\le x\end{cases}$

> We *can't* say $F(x)=\frac{1}{5}x,\quad 0\le x<4$ because X is not a continuous random variable. We *do* need to cover a domain of $-\infty$ to ∞ for the cumulative distribution function F.

1.c. (1 point) $V(Y)=5^2 * E(X)=50$

1.d. (1 point) $\binom{10}{8}=\frac{10!}{8!\,2!}=\frac{10*9*8!}{8!*2}=45$

2.a. (5 points)



tree diagram not necessary for points – provided for informational purposes

Use Bayes' Theorem.

$$P(\text{Bio}\mid B)=\frac{P(B\cap\text{Bio})}{P(B\cap\text{Bio})+P(B\cap\text{Phys})}=\frac{0.6*0.6}{0.6*0.6+0.4*0.5}$$

2.b. (5 points) since n = 7 > 5% of 100: hypergeometric probability

$$P(X<3)=P(X\le 2)=P(X=0)+P(X=1)+P(X=2)=\frac{\binom{3}{0}\binom{97}{7}+\binom{3}{1}\binom{97}{6}+\binom{3}{2}\binom{97}{5}}{\binom{100}{7}}$$

3.a. (5 points) A Poisson distribution takes on discrete values 0, 1, 2, …

$$P(X+Y\le 1)=P(X=0\cap Y=0)+P(X=1\cap Y=0)+P(X=0\cap Y=1)$$

$$=\left(\frac{2^0e^{-2}}{0!}*\frac{3^0e^{-3}}{0!}\right)+\left(\frac{2^1e^{-2}}{1!}*\frac{3^0e^{-3}}{0!}\right)+\left(\frac{2^0e^{-2}}{0!}*\frac{3^1e^{-3}}{1!}\right)$$

3.b. (5 points) for $1\le x\le 2,\ f(x)=F'(x)=\frac{3}{7}x^2;\ E(X)=\int_1^2 x*\left(\frac{3}{7}x^2\right)dx$

4.a. (6 points) $\mu=\int_{-\infty}^{\infty}x\left(ke^{-kx}\right)dx=\lim_{b\to\infty}\int_0^b kxe^{-kx}\,dx;\ dv=e^{-kx}\,dx,\ v=-\frac{1}{k}e^{-kx},\ u=kx,\ du=k\,dx;$

$$\int kxe^{-kx}\,dx=kx\left(-\frac{1}{k}e^{-kx}\right)-\int k\left(-\frac{1}{k}e^{-kx}\right)dx=-xe^{-kx}-\frac{1}{k}e^{-kx}$$

$$\lim_{b\to\infty}\left[-xe^{-kx}-\frac{1}{k}e^{-kx}\right]_0^b=\lim_{b\to\infty}\left[-be^{-kb}-\frac{1}{k}e^{-kb}\right]-\left[-(0)e^{-k(0)}-\frac{1}{k}e^{-k(0)}\right]=\frac{1}{k}$$

4.b. (6 points) $0.5 = \int_{-\infty}^{\tilde{\mu}} ke^{-kx}\ dx = \int_0^{\tilde{\mu}} ke^{-kx}\ dx = \left[-e^{-kx}\right]_0^{\tilde{\mu}} = -e^{-k\tilde{\mu}} + 1 = 0.5$

$$e^{-k\tilde{\mu}} = \frac{1}{2} \quad \Rightarrow \quad -k\tilde{\mu} = \ln\left(2^{-1}\right) \quad \Rightarrow \quad k\tilde{\mu} = \ln(2) \quad \Rightarrow \quad \tilde{\mu} = \frac{1}{k}\ln(2)$$

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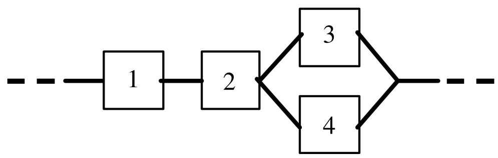
5. (8 points)



For each grouping: $P(\text{grouping works}) = P(\#1 \text{ works} \cap \#2 \text{ works} \cap [\#3 \text{ works} \cup \#4 \text{ works}])$

$$= P(\#1 \text{ works}) * P(\#2 \text{ works}) * [P(\#3 \text{ works}) + P(\#4 \text{ works}) - P(\#3 \text{ works} \cap \#4 \text{ works})]$$

$$= p * p * [p + p - p * p]$$

$$= 2p^3 - p^4$$

For the system: $P(\text{system works}) = P(\text{grouping 1 works} \cap \text{grouping 2 works} \cap \ldots \cap \text{grouping } k \text{ works})$

$$= \left(2p^3 - p^4\right) * \left(2p^3 - p^4\right) * \ldots * \left(2p^3 - p^4\right)$$

$$= \left(2p^3 - p^4\right)^k$$
