March 9, 2012 Stat 400 – EXAM 1 (2.1 – 4.2) [Pilachowski]

Follow directions carefully:

- Use exactly ONE answer sheet per question (use the back of the sheet if needed).
- Put your name and the question number on EACH page.
- You may use (your text or one 8.5x11 page) and a calculator.
- Show enough work that we can follow your thinking.

You must show all appropriate work in order to receive full credit for an answer.

- Before handing in your test: on your *first* answer sheet only, please copy the pledge and sign.

Answer question 1 on answer page 1.

1. (9 points) A discrete random variable takes on values 0, 1, 2, 3, and 4, with each value being equally likely.
   a. Find $V(X)$. (Do the calculations to get to an exact value numeric answer.)
   b. Find $F(x)$, the cumulative distribution function of X.
   c. Suppose random variable $Y = 5X + 2$. Find $V(Y)$. (Do the calculations to get to an exact value numeric answer.)

1.d. (1 point) First write out the calculation for $\binom{10}{8}$ using factorials, then compute to a numeric answer.

Answer question 2 on answer page 2.

2.a. (5 points) The table to the right gives conditional probabilities for grades received in two school subjects. 40% of the students are taking Physics and the rest are taking Biology. Set up the calculate needed to find the probability that a student was in Biology given that she got a B. (Do not compute.)

	A	B	C
Physics	0.2	0.5	0.3
Biology	0.3	0.6	0.1

2.b. (5 points) Of 100 electrical relays produced at a factory, 3 are defective. If a customer purchases 7 of these relays, what is the probability that less than 3 are defective? (Set up the calculation; do not compute.)

Answer question 3 on answer page 3.

3.a. (5 points) Suppose X and Y are independent random variables, each having a Poisson distribution with means 2 and 3, respectively. Find $P(X + Y \le 1)$. (Set up the calculation; do not compute.)

3.b. (5 points) A random variable X has cumulative distribution function $F(x) = \begin{cases} 0 & x < 1 \\ \frac{1}{7}\left(x^3 - 1\right) & 1 \le x < 2 \\ 1 & 2 \le x \end{cases}$. Set up the calculations necessary to find $E(X)$. (You do not have to do the computations.)

Answer question 4 on answer page 4.

4. (12 points) A random variable X has probability density function $f(x) = \begin{cases} ke^{-kx} & 0 \le x < \infty \\ 0 & \text{otherwise} \end{cases}$.

   a. Prove that the mean $= \frac{1}{k}$.

   b. Prove that the median $\frac{1}{k} * \ln(2)$.

Answer question 5 on answer page 5.

5. (8 points) Suppose independent components are arranged in groupings of four as pictured below. If a system consists of k of these groupings (connected end-to-end) and if an individual component has a probability of working $= p$, what is the probability of the system working?