1. (a) $E(X) = (1)\left(\frac{1}{4}\right) + (2)\left(\frac{1}{4}\right) + (3)\left(\frac{1}{4}\right) + (4)\left(\frac{1}{4}\right) = \left(\frac{1}{4}\right)(1+2+3+4) = \left(\frac{1}{4}\right)(10) = \frac{10}{4} = \frac{5}{2}$

1. (b) $E(X^2) = (1^2)\left(\frac{1}{4}\right) + (2^2)\left(\frac{1}{4}\right) + (3^2)\left(\frac{1}{4}\right) + (4^2)\left(\frac{1}{4}\right) = \left(\frac{1}{4}\right)(1^2 + 2^2 + 3^2 + 4^2) = \frac{30}{4} = \frac{15}{2}$

$$V(X) = E(X^2) - [E(X)]^2 = \frac{15}{2} - \left(\frac{5}{2}\right)^2 = \frac{30}{4} - \frac{25}{4} = \frac{5}{4}$$

1. (c) $F(x) = \begin{cases} 0, & x < 1 \\ \frac{1}{4}, & 1 \le x < 2 \\ \frac{2}{4} = \frac{1}{2}, & 2 \le x < 3 \\ \frac{3}{4}, & 3 \le x < 4 \\ 1, & x \ge 4 \end{cases}$

> We *can't* say $F(x) = \frac{1}{4}x, \quad 1 \le x < 4$ because X is not a continuous random variable. We *do* need to cover a domain of $-\infty$ to ∞ for the cumulative distribution function F.

(15 points)

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2. (a) $E(X) = \int_0^1 x * (n+1)x^n \, dx = \int_0^1 (n+1)x^{n+1} \, dx = \left[\frac{n+1}{n+2} x^{n+2}\right]_0^1 = \frac{n+1}{n+2}$

2. (b) $E(X) = \int_0^1 x^2 * (n+1)x^n \, dx = \int_0^1 (n+1)x^{n+2} \, dx = \left[\frac{n+1}{n+3} x^{n+3}\right]_0^1 = \frac{n+1}{n+3}$

$$V(X) = E(X^2) - [E(X)]^2 = \frac{n+1}{n+3} - \left(\frac{n+1}{n+2}\right)^2$$ stopping here was sufficient

$$V(X) = \frac{(n+1)(n+2)^2 - (n+1)^2(n+3)}{(n+3)(n+2)^2} = \frac{(n+1)(n^2+4n+4) - (n^2+2n+1)(n+3)}{(n+3)(n+2)^2}$$

$$= \frac{(n^3 + 4n^2 + 4n + n^2 + 4n + 4) - (n^3 + 2n^2 + n + 3n^2 + 6n + 3)}{(n+3)(n+2)^2}$$

$$= \frac{n^3 - n^3 + 4n^2 + n^2 - 2n^2 - 3n^2 + 4n + 4n - n - 6n + 4 - 3}{(n+3)(n+2)^2} = \frac{n+1}{(n+3)(n+2)^2}$$

2. (c) for $0 \le x \le 1$, $F(x) = \int_0^x (n+1)t^n \, dt = \left[t^{n+1}\right]_{t=0}^{t=x} = x^{n+1}$

$$F(x) = \begin{cases} 0, & x < 0 \\ x^{n+1}, & 0 \le x < 1 \\ 1, & x \ge 1 \end{cases}$$

2. (d) $F(x) = x^{n+1} = \frac{1}{2} \Rightarrow \tilde{\mu} = \left(\frac{1}{2}\right)^{\frac{1}{n+1}} = \sqrt[(n+1)]{\frac{1}{2}}$

> Note that the answers to both (d) and (e) are between 0 and 1, as they should be.

2. (e) $F(x) = x^{n+1} = \frac{3}{4} \Rightarrow 75^{th}$ percentile $= \left(\frac{3}{4}\right)^{\frac{1}{n+1}} = \sqrt[(n+1)]{\frac{3}{4}}$

(20 points)

3. (a) Define S = system works, $A_i = i^{\text{th}}$ component works ($1 \le i \le n$);
For the system to work, all components must work i.e. $P(S) = P(A_1 \cap A_1 \cap \ldots A_n)$.

Since A_i are independent events, $P(S) = P(A_1) * P(A_2) * \ldots P(A_n) = p * p * \ldots * p = p^n$.

3. (b) For the system to work, at least one component must work. This is the complement of "none of the components work". It is important to note that, like events A_i, events A_i' are independent.

$P(S) = 1 - P(S') = 1 - P(A_1' \cap A_2' \cap \ldots \cap A_n') = 1 - (1-p)^n$.

(5 points)

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4. for two children, S = {BB, BG, GB, GG};

$$P(\text{two girls given at least one is a girl}) = \frac{P(\text{older is a girl} \cap \text{younger is a girl})}{P(\text{at least one is a girl})} = \frac{P(\text{GG})}{P(\text{BG or GB or GG})} = \frac{1/4}{3/4} = \frac{1}{3}$$

(5 points)

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5. Each of 3 balls distinguishable by color is either "in box 1 therefore not in box 2" or "not in box 1 therefore in box 2": $2^3 = 8$ permutations.

Since they are all red, balls are not distinguishable. Box 1 has 0, 1, 2 or 3 balls: 4 combinations.

(5 points)