

J.Millson

1. Let  $X$  be a discrete random variable with probability mass function  $p$  given by the following table

x	1	2	3	4
P(X=x)	1/4	1/4	1/4	1/4

- (a) Find  $E(X)$ .
- (b) Find  $V(X)$ .
- (c) Find  $F(x)$ , the cumulative distribution function of  $X$ .

(15 points)

2. Let  $X$  be a continuous random variable with the probability density function

$$f(x) = \begin{cases} (n+1)x^n, & 0 \leq x \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

- (a) Find  $E(X)$ .
- (b) Find  $V(X)$ .
- (c) Find  $F(x)$ , the cumulative distribution function of  $X$ .
- (d) Find the median of  $X$ .
- (e) Find the 75-th percentile of  $X$ .

(Your answers to (a), (b), (d) and (e) should be functions of  $n$ . Your answer to (c) should be a function of  $x$  and  $n$ .)

(20 points)

3. (a) Suppose you have a system of  $n$ -components connected in in series . The components function independently and each component has probability  $p$  of working. What is the probability the system will work.

(b) Take everything the same as in part (a) except assume the components are arranged in parallel. What is the probability the system will work? (Hint: first compute the probability the system does not work.)

(5 points)

4. Suppose a friend of yours is a married woman with two children. You ask her “ do you have at least one girl child ” and she says “yes”. What is the probability both her children are girls.

(5 points)

5. Suppose you have a red ball, a blue ball and a white ball and you have two boxes. How many ways can you distribute the three balls into the two different boxes ( you are allowed to put all the balls into either of the two boxes)? How many ways are there if all the balls are red?

(5 points)