Stat 400 **MIDTERM 2** **April 21, 2011**
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1. Suppose X and Y are random variables defined on the same sample space with the following joint probability mass function.

X \ Y	0	1	2
0	1/4	1/4	1/8
1	1/4	0	1/8

(a) Compute the probability mass functions of the random variables X and Y.
(b) Are X and Y independent?
(c) Compute the probability mass function of the random variable $W = X + Y$.
(d) Compute the covariance $Cov(X, Y)$.
(e) Compute the correlation $\rho_{X,Y}$.
(20 points)

2. The ideal size of a first-year class at a particular college is 50 students. The college knows from past experience that on average 50% of those accepted for admission actually attend the college. The college uses a policy of accepting the applications of the top 100 students who apply.

(a) Write down a formula in terms of binomial coefficients and fractions for the probability that more than 50 students will attend the college.

(b) Using the normal distribution tables on the back write down a number that approximates the answer in (a).
Don't worry about the correction for continuity.
(10 points)

For the next problem you will need the fact that if X has exponential distribution with parameter λ then $P(X > t) = e^{-\lambda t}$.

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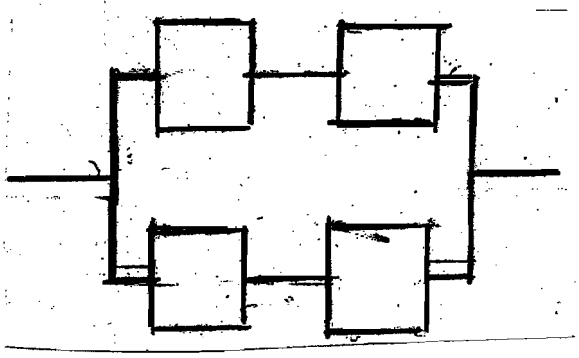
3. Suppose a system of four components is arranged as follows:



Assume the components function independently and that the lifetime of each component is exponentially distributed with parameter λ. Find the cumulative density function for the system lifetime.

(10 points)

4. You arrive at a bus stop at 10 o'clock, knowing that the bus will arrive at some time uniformly distributed between 10:00 and 10:30.

(a) What is the probability you will have to wait longer than 10 minutes?

(b) If at 10:15 the bus has not yet arrived , what is the probability that you will have to wait an additional 10 minutes?

(10 points)

Table A.3 Standard Normal Curve Areas *(cont.)* $\Phi(z) = P(Z \leq z)$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9278	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817