Stat 400 2010 Final

Solutions

1.(a) $E(X) = \int_0^1 x\,(n+1)x^n\,dx = (n+1)\int_0^1 x^{n+1}\,dx$

$= \frac{n+1}{n+2}$

(b) $E(X^2) = \int_0^1 x^2 (n+1)x^n\,dx = (n+1)\int_0^1 x^{n+2}\,dx$

$= \frac{n+1}{n+3}$

(c) $V(X) = \frac{n+1}{n+3} - \left(\frac{n+1}{n+2}\right)^2$

(d) $F(x) = \begin{cases} 0, & x < 0 \\ x^{n+1}, & 0 \le x \le 1 \\ 1, & 1 < x \end{cases}$

(e) $\tilde{\mu}$ is the solution of $x^{n+1} = \frac{1}{2}$

so $\tilde{\mu} = \left(\frac{1}{2}\right)^{\frac{1}{n+1}}$

(f) $\eta(.9)$ is the solution of $x^{n+1} = \frac{9}{10}$

so $\eta(.9) = \left(\frac{9}{10}\right)^{\frac{1}{n+1}}$

2(a) Let X = the number of passengers who get off at the first stop

I claim $X \sim Bin(n, \frac{1}{3})$.

Here is the relation with coin flipping.

Flip a coin = pick a passenger

Head (success) = the passenger you picked gets off the bus

$P(\text{success}) = \frac{1}{3}$

So $P(X=k) = \binom{n}{k}\left(\frac{1}{3}\right)^k\left(\frac{2}{3}\right)^{n-k}$

In particular $P(X=0) = \left(\frac{2}{3}\right)^n$

(b) Let S = today is Sunday

We want $P(S|X=0)$. By Bayes Theorem

$$P(S|X=0) = \frac{P(X=0|S)P(S)}{P(X=0|S)P(S) + P(X=0|S')P(S')}$$

Now if S holds then $n=2$ so $P(X=0|S) = \left(\frac{2}{3}\right)^2$

and if S' holds then $n=4$ so $P(X=0|S') = \left(\frac{2}{3}\right)^4$

Also $P(S) = \frac{1}{7}$, $P(S') = \frac{6}{7}$. So

$$P(S|X=0) = \frac{\left(\frac{2}{3}\right)^2\left(\frac{1}{7}\right)}{\left(\frac{2}{3}\right)^2\left(\frac{1}{7}\right) + \left(\frac{2}{3}\right)^4\left(\frac{6}{7}\right)} = \frac{\cancel{\frac{1}{7}}}{\cancel{\frac{1}{7}} + \left(\frac{2}{3}\right)^2\cancel{\left(\frac{6}{7}\right)}}$$

$$= \frac{1}{1 + \left(\frac{4}{\cancel{9}_3}\right)\cancel{6}_2} = \frac{1}{1+\frac{8}{3}} = \frac{3}{11}$$

3 (a) $\#(S) = (52)(51)(50)$

(b) $\#(A) = (13)(51)(50)$

$$P(A) = \frac{\#(A)}{\#(S)} = \frac{(13)\cancel{(51)}\cancel{(50)}}{(52)\cancel{(51)}\cancel{(50)}} = \frac{1}{4}$$

(c)

$\#(B) = (51)(50)(1)$

$$P(B) = \frac{\#(B)}{\#(S)} = \frac{\cancel{(51)}\cancel{(50)}(1)}{(52)\cancel{(51)}\cancel{(50)}} = \frac{1}{52}$$

(d) $\underbrace{(51)(50)\cdots(3)(2)(1)}_{\text{There are 51 numbers here}}(13) = (13)(51)!$

$$P\left(\substack{\text{last card}\\ \text{is a heart}}\right) = P(C) = \frac{\#(C)}{\#(S)} = \frac{(13)(51)!}{(52)!} = \frac{13}{52} = \frac{1}{4}$$

$= P(\text{first card is a heart})$

4 (a)

	−1	0	1
0	$\frac{1}{4}$	0	$\frac{1}{4}$
1	0	$\frac{1}{2}$	0

(b) This part was incorrectly posed. It belongs with a different part (a)

DON'T TRY TO DO IT

5. Just plug into the formula

$$\hat{p} = \frac{10}{100} = \frac{1}{10} \quad \text{so} \quad \hat{q} := 1 - \hat{p} = \frac{9}{10}$$

$$n = 100$$

Confidence level $= 100(1-\alpha)\% = 90\%$

so $1 - \alpha = .9$ so $\alpha = .1$

so $\alpha/2 = .05$

From the table $z_{.05} = 1.645$ so

$$I = \underbrace{\left(\frac{1}{10} - (1.645)\sqrt{\frac{\left(\frac{1}{10}\right)\left(\frac{9}{10}\right)}{100}},\ \frac{1}{10} + (1.645)\sqrt{\frac{\left(\frac{1}{10}\right)\left(\frac{9}{10}\right)}{100}} \right)}_{\text{This is good enough}}$$

$$\sqrt{\frac{\frac{1}{10}\frac{9}{10}}{100}} = \sqrt{\frac{9}{(100)^2}} = \frac{3}{100} = .03$$

$$(.03)(.1645) = .04935$$

$$I = (.05065,\ .14935)$$

We didn't cover 6.