MATH 437: FIRST HOMEWORK

Due Thursday, February 6, 2014.
The problem numbers and page numbers for these seven problems refer

(1) (Problem 2.6, p.9) Let

\[ D(x, y) := \begin{vmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} \end{vmatrix} \].

If for some point \((x_0, y_0)\), you know \(D(x_0, y_0) > 0\), then show
that the signs of

\[ \frac{\partial^2 f}{\partial x^2}(x_0, y_0), \quad \frac{\partial^2 f}{\partial y^2}(x_0, y_0) \]

agree. Find examples of such functions where \((x_0, y_0) = (0, 0)\).

(2) (Problem 2.11, p.13) The tangent lines to the graph \(z = f(x, y)\)
of a function \(\mathbb{R}^2 \xrightarrow{f} \mathbb{R}\) at a point \((x_0, y_0, f(x_0, y_0))\) correspond,
by horizontal projection

\[ \mathbb{R}^3 \xrightarrow{\Pi} \mathbb{R}^2 \]

\((x, y, z) \mapsto (x, y)\)
to lines \(\ell \subset \mathbb{R}^2\) through \((x_0, y_0)\). Define the slope of such a line
to be the slope of that line in the vertical plane \(\Pi^{-1}(\ell)\) which
projects to \(\ell\), where the \(z\)-coordinate is considered “vertical.”
Let \(f(x, y)\) be the function

\[ f(x, y) := \frac{1}{2}x^2 + 3xy \]

Find the largest slope of any tangent line to the graph \(z = f(x, y)\) of \(f(x, y)\) at the point \((1, 1)\).

\underline{Date:} January 30, 2014.
3. (Problem 2.18, p.16) Draw the curves given by the following parametrizations:
(a) \((t, t^2)\) where \(0 \leq t \leq 1\);
(b) \((t^2, t^3)\) where \(0 \leq t \leq 1\);
(c) \((2 \cos(t), 3 \sin(t))\) where \(0 \leq t \leq 2\pi\);
(d) \((\cos(2t), \sin(3t))\) where \(0 \leq t \leq 2\pi\);
(e) \((t \cos(t), t \sin(t))\) where \(0 \leq t \leq 2\pi\).

4. (Problem 2.20, p.17) Parametrize, in two different ways, the line segment connecting the points \((1, 1)\) and \((2, 5)\). Find a parametrization which has unit speed (that is, parametrize the curve by arc length).

5. (Problem 2.35, p.21) Consider the parametrization
\[\phi(r, \theta) = (r \cos(\theta), r \sin(\theta), \sqrt{r})\]
where \(0 \leq r \leq 2\) and \(0 \leq \theta \leq 2\pi\).
(a) Sketch the surface parametrized by \(\phi\).
(b) Find two tangent vectors to this surface at the point \((0, 1, 1)\).

6. (Problem 2.36, p. 22) Let \(B\) be the ball of radius 1 in \(\mathbb{R}^3\), that is, the set of points \((x, y, z)\) in \(\mathbb{R}^3\) satisfying \(x^2 + y^2 + z^2 \leq 1\).
(a) Use spherical coordinates to find a parametrization for \(B\).
(b) Parametrize the intersection of \(B\) with the first octant, that is, the region where all \(x, y, z\) are all positive.
(c) Parametrize the intersection of \(B\) with the octant where \(x, y, z\) are all negative.

7. (Problem 2.41, p. 23) Consider the following parametrization:
\[\psi(r, \theta) = (2r \cos(\theta), r \sin(\theta) + 1),\]
where \(0 \leq r \leq 1\) and \(0 \leq \theta \leq \pi\).
(a) Sketch the region of \(\mathbb{R}^2\) parametrized by \(\psi\).
(b) Find the vectors \(\partial \psi/\partial r\) and \(\partial \psi/\partial \theta\).
(c) Find the area of the parallelogram spanned by these two vectors.