(1) In the respective affine charts

\((x, y) \mapsto [x : y : 1]\),

\((y, z) \mapsto [1 : y : z]\),

\((x, z) \mapsto [x : 1 : z]\),

the conic

\(C := \{[X : Y : Z] \in \mathbb{P}^2 \mid XY + YZ + ZX = 0\}\).

is described in affine coordinates as, respectively:

\(xy + x + y = 0\),

\(yz + y + z = 0\)

\(xz + x + z = 0\).

(a) Compute the ideal points for these affine conics.

(b) What kinds of affine conics (circles, ellipses, parabolas, hyperbolas, degenerate conics) are these?

(c) Find an affine patch in which \(C\) is an ellipse.

(2) Compute the rotation of angle \(\theta\) about the line

\[\ell := \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + \mathbb{R} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.\]

(3) Let \(q\) be a quaternion. A square root of \(q\) is a quaternion \(p\) such that \(pp = q\) (quaternion multiplication). How many square roots does \(q\) have? (Like for real and complex numbers, the answer might depend on \(q\).)

(4) Compute \(\exp\begin{bmatrix} a & b \\ 0 & a \end{bmatrix}\).

(5) Graph the curve \(y = x^3\) in the affine plane. Compute its ideal points in \(\mathbb{P}^2\) (if any). For each ideal point \(p\), find an affine patch \(A\) containing \(p\) and graph the curve in \(A\).