Let $M$ be a connected topological (respectively smooth) manifold. (As usual, assume $M$ is Hausdorff and second countable.)

Prove or disprove each of the following statements:

- Let $x, y \in M$ and $x \neq y$. \exists an embedding $[0, 1] \xrightarrow{\gamma} M$
  with $\gamma(0) = x$ and $\gamma(1) = y$. If $M$ is smooth, $\gamma$ can be chosen to be a smooth embedding.

- $M$ is path-connected.

- Now suppose $x_1, \ldots, x_n, y_1, \ldots, y_n \in M$ are $2n$ distinct points. \exists an embedding $[0, 1] \times \{1, \ldots, n\} \xrightarrow{\gamma} M$
  with $\gamma(0, i) = x_i$
  $\gamma(1, i) = y_i$
  for $i = 1, \ldots, n$. If $M$ is smooth, $\gamma$ may be chosen to be smooth.

- The group of homeomorphisms (respectively diffeomorphisms) is $n$-point-transitive on $M$: that is, given two $n$-tuples $x_1, \ldots, x_n$, $y_1, \ldots, y_n$ of points on $M$, \exists a homeomorphism (respectively diffeomorphism) $M \xrightarrow{\phi} M$ such that $\phi(x_i) = y_i$
  for $i = 1, \ldots, n$.

- The preceding $n$-point homogeneity holds when $n = \infty$, provided that the subsets $\{x_1, x_2, \ldots\}$ and $\{y_1, y_2, \ldots\}$ are discrete.

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Date: 26 October 2006.