GEOMETRIC STRUCTURES ON MANIFOLDS
MATH 748G – SELECTED TOPICS IN GEOMETRY

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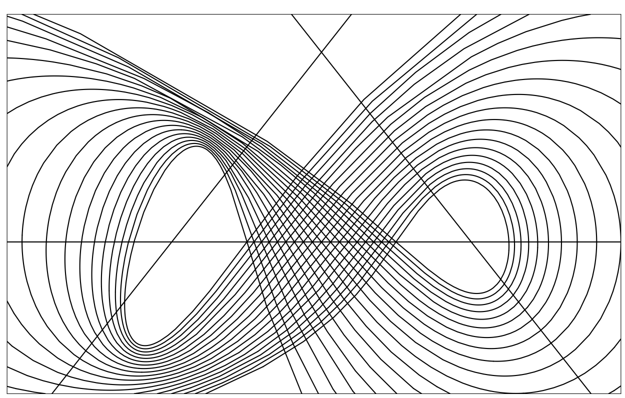
FALL 2007

Given a *topology,* a loose organization of points, can you rigidify it by geometry? A *geometry* here means a homogeneous space of a Lie group, with such quantitative notions as distance, angle, parallelism, linearity, circularity, and so on. Even geometries lacking an invariant notion of distance are rich and fascinating. A *locally homogeneous geometric structure* (in the sense of Ehresmann and Thurston) is defined by a system of local coordinate systems on the topology, taking values in the geometry. Different local coordinates on overlapping patches relate by geometric automorphisms

For example the sphere S^n admits no structure modeled on Euclidean geometry $\mathbb{R}^n$: every world atlas must be metrically inaccurate somewhere. This subject naturally leads into the theory of moduli of representations of fundamental groups of surfaces. Full of rich examples, it relates to Differential Geometry, Geometric Topology, Algebraic Geometry, Lie Groups, Dynamical Systems, and PDE.

We will loosely follow selected publications and preprints, my notes *Projective geometry of manifolds,* (which I am expanding into a book) and Thurston's monograph, *Three-dimensional geometry and topology.* The class meets Monday-Wednesday-Fridays at 12:00.



A developing map for an $\mathbb{RP}^2$-structure on the 2-torus which is not a covering-space onto its image