1. Name the only two Mathematics Departments in the USA with all three of the following:
   i. A Ph.D Graduate with a Nobel prize.
   ii. An undergraduate Math major who went on to win a Fields Medal.
   iii. A Fields Medalist on the faculty.

2. Let
   \[ A = \begin{pmatrix} 3 & 6 & 7 \\ 3 & 3 & 7 \\ 5 & 6 & 5 \end{pmatrix} \]

   is \( (1, -2, 1)^T \) an eigenvector of \( A \)? If so, find the eigenvalue.

3. Let
   \[ B = \begin{pmatrix} 2 & 3 & 3 \\ 12 & 5 & 6 \\ -27 & -15 & -16 \end{pmatrix} \]

   Given that \( (3 + 2i, 5 - i, -13)^T \) is an eigenvector of \( B \) corresponding to the eigenvalue \( \lambda = -4 + 3i \), find another eigenvalue of \( B \) and a corresponding eigenvector.

4. Let \( A \) be a \( 2 \times 2 \) matrix whose eigenvalues are \( \lambda_1 = -1 \) and \( \lambda_2 = 2 \) with corresponding eigenvectors \( v_1 = (1, 1)^T \) and \( v_2 = (2, 3)^T \). Solve the initial value problem \( x' = Ax, \ x(0) = (5, 2)^T \).

5. Let \( A \) be as in problem 4. What is \( A \)?

6. Let \( u = (1, 1, 1, 1)^T, \ v = (1, 7, 1, 7)^T, \ W = \text{Span}\{u, v\} \).
   (a) Calculate \( ||v||, \ \text{dist}(u, v) \), the projection of \( v \) onto \( u \) and the unit vector in the direction of \( u \).
   (b) Apply the Gram-Schmidt process to \( \{u, v\} \) to obtain an orthonormal basis for \( W \).
   (c) Let \( y = (3, 2, -1, 2)^T \). Find \( z \), the vector in \( W \) which is closest to \( y \).