In this assignment we will investigate curves in two and three dimensions. For many of the problems, the output is a picture. Be sure to title the pictures using the \texttt{title} command.

1. We will first graph the curve $\mathbf{r}(t) = (\sin t - t \cos t) \mathbf{i} + (\cos t + t \sin t) \mathbf{j}$ for $0 \leq t \leq 4\pi$ together with some velocity vectors. So run the following script. (Note: a line starting with \% is a comment and needn’t be typed in.)

   \begin{verbatim}
   t=linspace(0,4*pi,121);
   x=sin(t)-t.*cos(t);
   y=cos(t)+t.*sin(t);
   plot(x,y)
   hold on
   % Add the velocity vectors
   for s=linspace(.16*pi,4*pi,24);
   r=[sin(s)-s.*cos(s),cos(s)+s.*sin(s)];
   u=[s.*sin(s),s.*cos(s)];
   arrow(r,u,'r')
   end
   hold off
   \end{verbatim}

2. Repeat problem 1 for the parabola $\mathbf{r}(t) = t \mathbf{i} + t^2 \mathbf{j}$ for $-2 \leq t \leq 2$.

3. Now we’ll look at some space curves. We consider the helix $\mathbf{r}(t) = \cos t \mathbf{i} + \sin t \mathbf{j} + t/(2\pi) \mathbf{k}$ for $0 \leq t \leq 4\pi$. Run the following script.

   \begin{verbatim}
   t=linspace(0,4*pi,101);
   x=@(t) cos(t);
   y=@(t) sin(t);
   z=@(t) t/(2*pi);
   plot3(x(t),y(t),z(t))
   hold on
   % Add the velocity vectors.
   for s=linspace(0,4*pi,17);
   p=[x(s),y(s),z(s)];
   v=[-sin(s),cos(s),1/(2*pi)];
   arrow3(p,v,'r')
   end
   view(135,40)
   hold off
   \end{verbatim}

4. Repeat problem 3 for the curve $\mathbf{r}(t) = t \mathbf{i} + t^2/2 \mathbf{j} + t^3/3 \mathbf{k}$ for $-2 \leq t \leq 2$. If you follow the model of problem 3 remember to write “\texttt{y=@(t) t.^2/2}” etc.
5. We can also exhibit the Frenet frame. So execute the first 5 lines in problem 3. Then do

\begin{verbatim}
axis equal
hold on
frenet(x,y,z)
\end{verbatim}

At this point you will get a prompt asking you to enter a value of $t$. Take $t = 0, 0.8\pi, 2.2\pi, 3.6\pi$. Then do

\begin{verbatim}
view(135,40)
\end{verbatim}

to rotate the figure. The result should be very pretty.

Now we’ll investigate arclength. Let the curve $C$ be parameterized by

$$r(t) = ti + t^2/2j + t^3/3k, \quad 0 \leq t \leq 2$$

6. First, we approximate the length of $C$ by making a polygonal approximation.

\begin{verbatim}
t=0:.02:2;
x=t; y=t.^2/2; z=t.^3/3;
sum=0;
for j=1:100
    dx=x(j+1)-x(j);
    dy=y(j+1)-y(j);
    dz=z(j+1)-z(j);
    dr=[dx, dy, dz];
    sum=sum+norm(dr);
end
disp('Length of the polygonal approx. using 100 segments')
sum
\end{verbatim}

7. Next we will use the numerical integrator \texttt{quadl}. We have

$$\|r'(t)\| = \sqrt{1 + t^2 + t^4}.$$ 

The arclength integral cannot be computed by hand. So do

\begin{verbatim}
speed=@(t) sqrt(1+t.^2+t.^4)
s=quadl(speed,0,2)
\end{verbatim}

Compare your answer with the answer for problem 6.

8. Let the curve $C$ be parameterized by

$$P(t) = (1 - \cos t)i + (1 + 2t + t^2)j.$$ 

(a) Calculate by hand a tangent vector to the curve at $P(0)$. 

(b) Use MATLAB to compute secant vectors \((\mathbf{P}(t) - \mathbf{P}(0))/t\) for \(t = .2, .1, .05\). The error in the secant approximation is

\[
\frac{\mathbf{P}(t) - \mathbf{P}(0)}{t} - \mathbf{P}'(0).
\]

By what factor is the error in each component decreased when \(t\) is cut in half?

(c) Plot the curve \(C\) for \(0 \leq t \leq 1\) and use the arrow feature to plot each of these secant vectors as well as the tangent vector computed in part (a). Attach each of these vectors to the point \(\mathbf{P}(0) = (0, 1)\).