1. Consider the expression
\[ \frac{1}{1-x} - \frac{1}{1+x}, \]
assuming \( x \neq \pm 1 \).
(a) For what range of values of \( x \) is it difficult to compute this expression accurately in floating-point arithmetic?
(b) Give a rearrangement of the terms such that, for the range of \( x \) in part (a), the computation is more accurate in floating-point arithmetic.

2. Assume a decimal (base 10) floating point system having machine precision \( \epsilon_{mach} = 10^{-5} \) and an exponent range of \( \pm 20 \). What is the result of each of the following floating-point operations
(a) \( 1 + 10^{-7} \)
(b) \( 1 + 10^3 \)
(c) \( 1 + 10^7 \)
(d) \( 10^{10} + 10^3 \)
(e) \( 10^{10}/10^{-15} \)
(f) \( 10^{-10} \times 10^{-15} \)

3. Let
\[ A = \begin{pmatrix} 4 & -2 \\ -2 & 2 \end{pmatrix} \]
(a) Find a lower triangular matrix \( L \) such that \( A = LL^T \) (Choleski factorization).
(b) Let \( \mathbf{b} = (10, -4)^T \). Use the Choleski factorization to solve \( Ax = b \) by forward elimination and back substitution.

4. In \( \mathbb{R}^2 \), is it possible to have two vectors \( x \) and \( y \) such that \( \| x \|_1 > \| y \|_1 \) but \( \| x \|_\infty < \| y \|_\infty \) ? If so, give an example.

5. (a) How is the condition number of a matrix \( A \) defined for a given matrix norm?
(b) How is the condition number used in estimating the accuracy of a computed solution to a linear system \( Ax = b \) ?

6. Given the three data points \((-1,2), (0,1), (1,2)\) Find the interpolating quadratic:
(a) in the form \( ax^2 + bx + c \) by solving a system of linear equations.
(b) in the Lagrange form
(c) in a Newton form.
Show that the three representations give the same polynomial.

7. Let
\[ s(x) = \begin{cases} 
  x + 1 & -2 \leq x \leq -1, \\
  x^3 - 2x - 1 & -1 \leq x \leq 1, \\
  x - 3 & 1 \leq x \leq 2. 
\end{cases} \]
Is \( s(x) \) a natural cubic spline? Explain.