

1. Exercise 1, p.134, **Stoer and Bulirsch**.
2. Exercise 2, p.134, **Stoer and Bulirsch**. Give both the Lagrange form and a Newton form.
3. Exercise 4, p.134, **Stoer and Bulirsch**.
4. Exercise 22, p.140, **Stoer and Bulirsch**.
5. Exercise 24, p.140, **Stoer and Bulirsch**.
6. Exercise 29, p.141, **Stoer and Bulirsch**.
7. Modify the proof given in class for the existence of a unique natural cubic spline interpolating given data points to show the existence of a unique “not-a-knot” spline.
8. (MATLAB) Observed values for the thrust (T) versus time (t) curve of a model rocket are

t	0.00	0.05	0.10	0.15	0.20
T	0.0	1.0	5.0	15.0	33.5
t	0.30	0.40	0.50	0.60	0.70
T	33.0	16.5	16.0	16.0	16.0
t	0.80	0.85	0.90	0.95	1.00
T	16.0	16.0	6.0	2.0	0.0

- (a) Use the MATLAB functions POLYFIT and POLYVAL to find and plot the 14th degree polynomial interpolating this data.
 - (b) Use the MATLAB function SPLINE to find and plot the cubic spline interpolating the data.
 - (c) Which function do you think does a better job of interpolating the data ? Why ? Suppose we also observe $T(0.25) = 38.0$, $T(0.65) = 16.0$. What values do the interpolation functions give in each case ? Compare the results with the observed values.
9. Let $\{x_0, x_1, \dots, x_n\}$ be $n + 1$ distinct real numbers. We define the Vandemonde matrix V_n to be

$$V_n = \begin{pmatrix} 1 & x_0 & x_0^2 & \cdots & x_0^n \\ 1 & x_1 & x_1^2 & \cdots & x_1^n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_n & x_n^2 & \cdots & x_n^n \end{pmatrix}$$

Use induction to prove

$$\det(V_n) = \prod_{i,j=0,\dots,n, i>j} (x_i - x_j).$$