1. (25 points)
(a) (15 points) Assume a decimal (base 10) floating point system having machine epsilon $\epsilon_{mach} = 5 \times 10^{-6}$ and an exponent range of $\pm 20$. What is the result of each of the following floating-point operations?

  (i) $1 + 10^{-7}$
  (ii) $1 + 10^{3}$
  (iii) $1 + 10^{7}$
  (iv) $10^{10} + 10^{3}$
  (v) $10^{10}/10^{-15}$
  (vi) $10^{-10} \times 10^{-15}$

(b) (10 points) How can values of $f(x) = \sqrt{x+4} - 2$ be computed accurately (avoiding loss of significance) when $x$ is small?

2. (40 points) Let $f(x) = x^3 + 2x - 7$.
(a) (3 points) Show there is a number $\alpha$ with $1 < \alpha < 2$ such that $f(\alpha) = 0$. (“This is clear from looking at the graph of $f$ on my graphing calculator” is not an acceptable answer.)

(b) (10 points) Use the bisection method with the initial interval $[1, 2]$ to approximate $\alpha$ with error less than $\frac{1}{16}$.

(c) (10 points) Let $x_0 = 1.5$. Use Newton’s method twice to compute new approximations to $\alpha, x_1$ and $x_2$.

(d) (10 points) Let $x_0 = 1, x_1 = 2$. Use the secant method twice to compute new approximations to $\alpha, x_2$ and $x_3$.

(e) (7 points) Will the iteration scheme

$$x_{n+1} = \frac{1}{2}(7 - x_n^3), \quad x_0 = 1.5$$

converge to $\alpha$? Explain.

3. (35 points)
(a) (25 points) Let

$$A = \begin{pmatrix} 1 & 1 & -2 \\ 1 & 2 & -2 \\ -2 & 1 & 1 \end{pmatrix}$$

(i) (15 points) Write $A = LU$ where $L$ is lower triangular and $U$ is upper triangular.

(ii) (10 points) Use the decomposition of part (i) to solve $Ax = b$ where $b = (1, 0, 1)'$ by forward elimination and back substitution.

(b) (10 points) Suppose you enter the vectors $a = [1 2 3]$ and $b = [4 5 6]$ into MATLAB. What will be the result if you enter the following commands?

(i) $a * b$
(ii) $a.* b$
(iii) $a * b'$