1. In a standard shell and tube heat exchanger hot vapor condenses on the tube, maintaining a constant temperature $T_s$. If the input is at temperature $T_1$ and the output must be at temperature $T_2$, then the length of tube required is given by

$$L = \frac{m}{\pi D} \int_{T_1}^{T_2} \frac{c_\rho dT}{h(T_s - T)}.$$  

(All quantities must be in consistent units.) Here $T$ is the temperature in °F.  
$T_1 = 0°F$ is the inlet temperature.  
$T_2 = 180°F$ is the desired outlet temperature.  
$T_s = 250°F$ is the condensate temperature.  
$m$ is the fluid flow rate $= 45,000$ lb/hr.  
$D$ is the diameter of the tube $= 1.032$ in.  
$c_\rho$ is the specific heat of the fluid $= (0.53 + 0.00065T)\text{BTU/(lb°F)}$.  
$h$ is the local heat transfer coefficient $= \frac{0.023k}{D} \left(\frac{4m}{\pi D \mu}\right)^{0.8} \left(\frac{\mu c_\rho}{k}\right)^{0.4}$.  
$k$ is the thermal conductivity of the fluid $= 0.153\text{BTU/(hr ft°F)}$.  
$\mu$ is the viscosity of the fluid and has units lb/(ft hr). $\mu$ varies with temperature so that

<table>
<thead>
<tr>
<th>$T$</th>
<th>0</th>
<th>50</th>
<th>100</th>
<th>150</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>242</td>
<td>82.1</td>
<td>30.5</td>
<td>12.6</td>
<td>5.57</td>
</tr>
</tbody>
</table>

Use spline interpolation to define $\mu$ for other values of $T$ and calculate the required length of the heat exchanger.  
You will need to use the MATLAB functions SPLINE and QUAD (or QUADL). The answer is about 158.7 ft.

2. Consider the boundary value problem

$$y'' + \frac{1}{t} y' - \frac{1}{4t^2} y = \frac{3}{4t}, \quad 1 \leq t \leq 2, \quad y(1) = 2, \quad y(2) = 2.$$  

(a) Show that $y = 2t^{-1/2} - t^{1/2} + t$ is the solution. (i.e. $y$ satisfies the equation and the boundary conditions.)

(b) Solve the problem by finite differences. Use central difference approximations for $y'$ and $y''$. If you have a tridiagonal solver, use it. Otherwise, just use the backslash operator. Take $h = 0.1, h = 0.05$ and $h = 0.025$. Plot your solutions along with the exact solution as well as the difference between the exact and approximate solutions. The only output we want are the pictures.