1. Let 

\[ A = \begin{pmatrix} 9 & -3 \\ -3 & 5 \end{pmatrix} \]

(a) Compute the Choleski factorization of \( A = LL^T \) with \( L \) lower triangular) and use it to solve \( Ax = b \) with \( b = (9, 9)^T \).
(b) Find an approximate solution to \( Ax = b \) by doing two Jacobi iterations starting at \( x^{(0)} = (1, 2)^T \).
(c) Find an approximate solution to \( Ax = b \) by doing two Gauss-Seidel iterations starting at \( x^{(0)} = (1, 2)^T \).
(d) Which method gives a better approximation to the exact solution?

2. Consider the fixed point iteration \( x_{n+1} = g(x_n) \) where

\[ g(x) = \frac{x}{2} + \frac{1}{x}. \]

(a) Show that \( g([1, 2]) \subset [1, 2] \)
(b) Show that \( g \) has a unique fixed point \( \alpha \in [1, 2] \) and that given any \( x_0 \in [1, 2] \) the iterations will converge to \( \alpha \).
(c) Calculate \( \alpha \) explicitly.

3. (a) Find the node \( x_1 \) and the weight \( w_1 \) such that the integration rule

\[ \int_0^1 \sqrt{x} f(x) \, dx \approx w_1 f(x_1) \]

is exact if \( f(x) \) is any linear polynomial. (This is not a hard computation.)
(b) Use this rule to compute an approximation to

\[ \int_0^1 \sqrt{x} e^{-x} \, dx = 0.3789447. \]

4. Given the initial value problem

\[ \frac{dy}{dt} = t + y, \quad y(0) = 1. \]

Find approximate values of \( y(0.2) \) by using
(a) Two Euler steps with \( h = 0.1 \).
(b) One improved Euler step with \( h = 0.2 \).
In each case compare your answer with the exact solution \( y = 2e^t - t - 1. \)