1. Write a MATLAB program to evaluate $I = \int_a^b f(x) \, dx$ using the trapezoidal rule with $n$ subdivisions, calling the result $I_n$. Use the program to calculate the following integrals with $n = 2, 4, 8, 16, \ldots, 512$.

   (a) $\int_0^1 x^4 \arctan(x) \, dx$  
   (b) $\int_0^1 x^{2/3} \, dx$

   The exact value of the integral in (a) is $\frac{\pi+1-2\ln(2)}{20}$. Try to arrange your work so that you never compute the value of the integrand at any point more than once. Analyze empirically the rate of convergence of $I_n$ to $I$ by calculating the ratios

   $$R_n = \frac{I_{2n} - I_n}{I_{4n} - I_{2n}} \quad \text{and} \quad p_n = \frac{\log(R_n)}{\log(2)}$$

   In part (b) compute the extrapolated approximation to $I$,

   $$I \approx I_{4n} - \frac{(I_{4n} - I_{2n})^2}{(I_{4n} - I_{2n}) - (I_{2n} - I_n)}$$

   for $n = 128$.

2. Repeat problem 1 using Simpson’s rule.

3. Apply the corrected trapezoidal rule to the integral in problem 1(a). Compare the results with those of problem 2 for Simpson’s rule.

4. Use Gauss-Legendre integration with $n = 2, 4, 8$ nodes to the integrals of problem 1. Compare the results with those for the trapezoidal and Simpson methods.

5. Find approximate values of the integrals in problem 1 by computing the Romberg integral $I_{32}^{(5)}$ where $I_{32}^{(0)}$ is the $n$-panel trapezoid approximation and

   $$I_{32}^{(k)} = \frac{4^k I_{32}^{(k-1)} - I_{32}^{(k-1)}_{n/2}}{4^k - 1}$$

   for $n$ divisible by $2^k$.

6. Use the MATLAB function QUADL to find approximate values of the integrals 1(a) and 1(b).

7. Ex. 5.1, p.183, Shampine, Allen & Preuss.

8. The 10 point Newton-Cotes integration rule on $[0, 1]$ is

   $$\int_0^1 f(x) \, dx \approx \sum_{i=0}^{9} w_i f\left(\frac{i}{9}\right)$$
with the $w_i$ determined by requiring that the rule be exact for $f(x) = 1, x, x^2, \ldots x^9$.

(a) Use MATLAB to find the weights $w_i$.
(b) Apply the rule to the integrals in 1(a) and 1(b). Note the errors.

9. We wish to estimate the value of

$$I = \int_0^\infty x^{3/2}e^{-x}dx = \frac{3}{4}\sqrt{\pi}$$

(a) Truncate the integral and use QUAD on the finite part.
(b) Try the transformation $x = -\ln t$ on this integral and use QUADL on the new integral. (QUADL will complain but will do it).
(c) Use the 2, 4 and 8 point Gauss-Laguerre rules to estimate the integral. Compare your results with parts (a) and (b) above.

10. In a standard shell and heat exchanger hot vapor condenses on the tube, maintaining a constant temperature $T_s$. If the input is at temperature $T_1$ and the output must be at temperature $T_2$, then the length of tube required is given by

$$L = \frac{m}{\pi D} \int_{T_1}^{T_2} \frac{c_p dT}{h(T_s - T)}.$$ 

(All quantities must be in consistent units.) Here $T$ is the temperature in °F.
$T_1 = 60^\circ$F is the inlet temperature.
$T_2 = 500^\circ$F is the desired outlet temperature.
$T_s = 550^\circ$F is the condensate temperature.
$m$ is the fluid flow rate = 22.5 lb/hr.
$D$ is the diameter of the tube = 0.495 in.
$c_p$ is the specific heat of the fluid = 0.251 + 3.46 $\times$ 10^{-5}T $-$ \frac{14,400}{(T+460)^2}/BTU/(lb\text{ °F}).
$h$ is the local heat transfer coefficient = $\frac{0.023k}{D}(\frac{4m}{\pi D\mu})^{0.8}(\frac{\mu c_p}{k})^{0.4}$.
$\mu$ is the viscosity of the fluid = 0.0332($\frac{T+460}{460}$)$^{0.935}$ lb/(ft hr).
$k$ is the thermal conductivity of the fluid and has the units BTU/(hr ft °F). $k$ varies with temperature so that

<table>
<thead>
<tr>
<th>$T$</th>
<th>0</th>
<th>200</th>
<th>300</th>
<th>400</th>
<th>500</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td>.0076</td>
<td>.013</td>
<td>.0157</td>
<td>.0183</td>
<td>.0209</td>
</tr>
</tbody>
</table>

Use spline interpolation to define $k$ for other values of $T$ and calculate the required length of the heat exchanger.
You will need to use the MATLAB functions SPLINE and QUADL.