1. Write a MATLAB program to evaluate $I = \int_a^b f(x) \, dx$ using the trapezoidal rule with $n$ subdivisions, calling the result $I_n$. Use the program to calculate the following integrals with $n = 2, 4, 8, 16, \ldots, 512$.

   (a) $\int_0^1 x^4 \arctan(x) \, dx$ \quad (b) $\int_0^1 x^{2/3} \, dx$

The exact value of the integral in (a) is $\frac{1}{20} \left( \frac{\pi^4}{32} - 2 \ln(2) \right)$. Try to arrange your work so that you never compute the value of the integrand at any point more than once. Analyze empirically the rate of convergence of $I_n$ to $I$ by calculating the ratios

$$R_n = \frac{I_{2n} - I_n}{I_{4n} - I_{2n}}$$

and

$$p_n = \frac{\log(R_n)}{\log(2)}$$

In part (b) compute the extrapolated approximation to $I$,

$$\tilde{I} = I_{4n} - \frac{(I_{4n} - I_{2n})^2}{(I_{4n} - I_{2n}) - (I_{2n} - I_n)}$$

for $n = 128$.

2. Repeat problem 1 using Simpson’s rule.

3. Apply the corrected trapezoidal rule to the integral in problem 1(a). Compare the results with those of problem 2 for Simpson’s rule.

4. Use Gauss-Legendre integration with $n = 2, 4, 8$ nodes to the integrals of problem 1. Compare the results with those for the trapezoidal and Simpson methods.

5. Find approximate values of the integrals in problem 1 by computing the Romberg integral $I_n^{(5)}$ where $I_n^{(0)}$ is the $n$-panel trapezoid approximation and

$$I_n^{(k)} = \frac{4^k I_n^{(k-1)} - I_n^{(k-1)}}{4^k - 1}$$

for $n$ divisible by $2^k$.

6. Use the MATLAB function QUADL to find approximate values of the integrals 1(a) and 1(b).

7. Ex. 5.1, p.183, Shampine, Allen & Preuss.

8. The 10 point Newton-Cotes integration rule on $[0, 1]$ is

$$\int_0^1 f(x) \, dx \approx \sum_{i=0}^{9} w_i f\left(\frac{i}{9}\right)$$
with the $w_i$ determined by requiring that the rule be exact for $f(x) = 1, x, x^2, \ldots, x^9$.

(a) Use MATLAB to find the weights $w_i$.

(b) Apply the rule to the integrals in 1(a) and 1(b). Note the errors.

9. We wish to estimate the value of

$$I = \int_0^\infty x^{3/2} e^{-x} \, dx = \frac{3}{4} \sqrt{\pi}$$

(a) Truncate the integral and use QUADL on the finite part.

(b) Try the transformation $x = -\ln t$ on this integral and use QUADL on the new integral. (QUADL will complain but will do it.)

(c) Use the 2, 4 and 8 point Gauss-Laguerre rules to estimate the integral. compare your results with parts (a) and (b) above.

10. In a standard shell and heat exchanger hot vapor condenses on the tube, maintaining a constant temperature $T_s$. If the input is at temperature $T_1$ and the output must be at temperature $T_2$, then the length of tube required is given by

$$L = \frac{m}{\pi D} \int_{T_1}^{T_2} \frac{c_p dT}{h(T_s - T)}.$$  

(All quantities must be in consistent units.) Here $T$ is the temperature in °F.

$T_1 = 0°F$ is the inlet temperature.

$T_2 = 180°F$ is the desired outlet temperature.

$T_s = 250°F$ is the condensate temperature.

$m$ is the fluid flow rate = 45,000 lb/hr.

$D$ is the diameter of the tube = 1.032 in.

$c_p$ is the specific heat of the fluid = \((0.53 + 0.00065T) \text{ BTU/(lb°F)}\).

$h$ is the local heat transfer coefficient = \(\frac{0.023k}{\pi D \mu} \left( \frac{4m}{D \mu} \right)^{0.8} \left( \frac{\mu c_p}{k} \right)^{0.4}\).

$k$ is the thermal conductivity of the fluid = \(0.153 \text{ BTU/(hr ft°F)}\).

$\mu$ is the viscosity of the fluid and has units lb/(ft hr). $\mu$ varies with temperature so that

\[
\begin{array}{cccccc}
T & 0 & 50 & 100 & 150 & 200 \\
\mu & 242 & 82.1 & 30.5 & 12.6 & 5.57
\end{array}
\]

Use spline interpolation to define $\mu$ for other values of $T$ and calculate the required length of the heat exchanger.

You will need to use the MATLAB functions SPLINE and QUADL. The answer is about 158.7 ft.