Geometry, Fall 2012

University of Maryland, Department of Mathematics course 430

HW 1: (due by September 13, in class.)

Please write-up your own solutions to problems in an organized and neat fashion. If collaborating in the problem solving process please write the names of the people with whom you collaborated next to each solution.

0. Prove the following: All points such that the segments of the tangent lines drawn from these points to two given non-concentric circles have equal lengths lie on one straight line. (Hint: you may use Cartesian coordinates, for instance.)

1. Find all of the flaws in the following argument. This is a good exercise in logic and an example of why you shouldn't depend too heavily on diagrams. Hint: as you read through the supposed proof, draw your own diagram to become familiar with the labeling and to find the flaws.

Proposition: Every triangle is isosceles.

"Proof:" Let $\triangle ABC$ be a triangle. We will show $AB \cong AC$. Construct the bisector of angle $\angle BAC$, and the perpendicular bisector of the side BC. Now we have four cases.

Case 1. The bisector of $\angle BAC$ and the perpendicular bisector of the side BC are identical lines or parallel lines. Being identical or parallel, we must have that the bisector of $\angle BAC$ is perpendicular to the side BC and hence is an altitude. The two triangles into which the bisector divides $\triangle ABC$ are congruent by the Angle-Side-Angle theorem (Euclid I.26). For instance, take the side that they share, the bisected angle, and the right angles formed by the altitude. Since these two triangles are congruent, $AB \cong AC$.

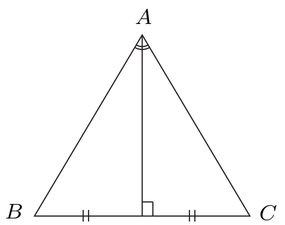
Now suppose the bisector of $\angle BAC$ is not parallel or identical to the perpendicular bisector of the line between BC. Then these lines meet in a point D. Now one of three cases occurs:
Case 2. D is inside the triangle.
Case 3. D is on the side BC.
Case 4. D is outside the triangle.

For each of these cases, construct DE perpendicular to AB and DF perpendicular to AC. In cases 2 and 4, draw the segments DB and DC as well. (In case 3, these are part of the triangle so they are already drawn). Now in each of the three cases, the argument proceeds as follows:

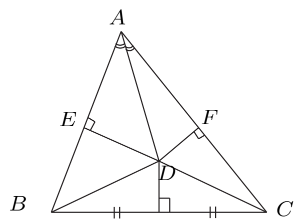
$DE \cong DF$ becaues all points on an angle bisector are equidistant from the sides of the angle. $\angle DEA$ and $\angle DFA$ are right angles, so $\triangle ADE \cong \triangle ADF$ by the right-angle-side-side theorem (Euclid III.14). Therefore $AE \cong AF$.

Now $DB \cong DC$ because all points on the perpendicular bisector of BC are equidistance from the points B and C. Also $DE \cong DF$ as we stated before, and $\angle DEB$ and $\angle DFC$ are right angles, so again using the right-angle-side-side theorem, $\triangle DEB \cong \triangle DFC$. Hence $FC \cong BE$.

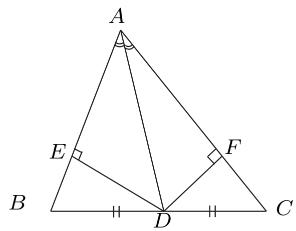
We now have shown $AE \cong AF$ and $FC \cong BE$. It follows that $AB \cong AC$ by addition in cases 2 and 3, and by subtraction in case 4. Since $AB \cong AC$, the triangle $\triangle ABC$ is isosceles.



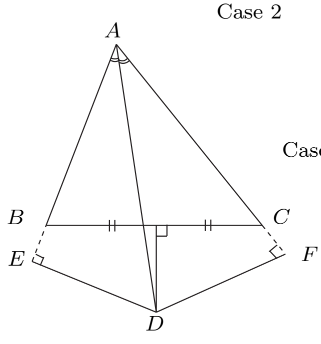
Case 1



Case 2



Case 3

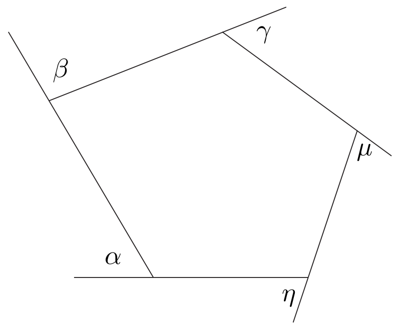


Case 4

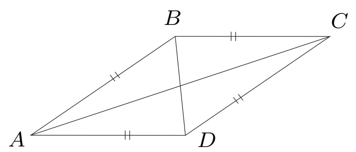
Read the Definitions, Postulates, Common Notions and Propositions I.1-I.32 in Book I of the Elements.
http://aleph0.clarku.edu/~djoyce/java/elements/bookI/bookI.html
For problems 2-4 below, you may use any of the these definitions, postulates, and propositions. Clearly state each proposition that you use to justify the steps in your proof.

2. (Stillwell Exercise 1.3.3) Construct a square (see Definition 22) on a given line segment.

3. Prove the exterior angles of a pentagon add up to four right angles.



4. A rhombus is a figure with four equal sides. Show that the diagonals of a rhombus meet at right angles. As a corollary, show that it follows that the four triangles thus formed are congruent to each other.



5. Given an angle at a point A and given another segment BC, construct a point D so that the angle $\angle DBC$ equals the given angle at A.